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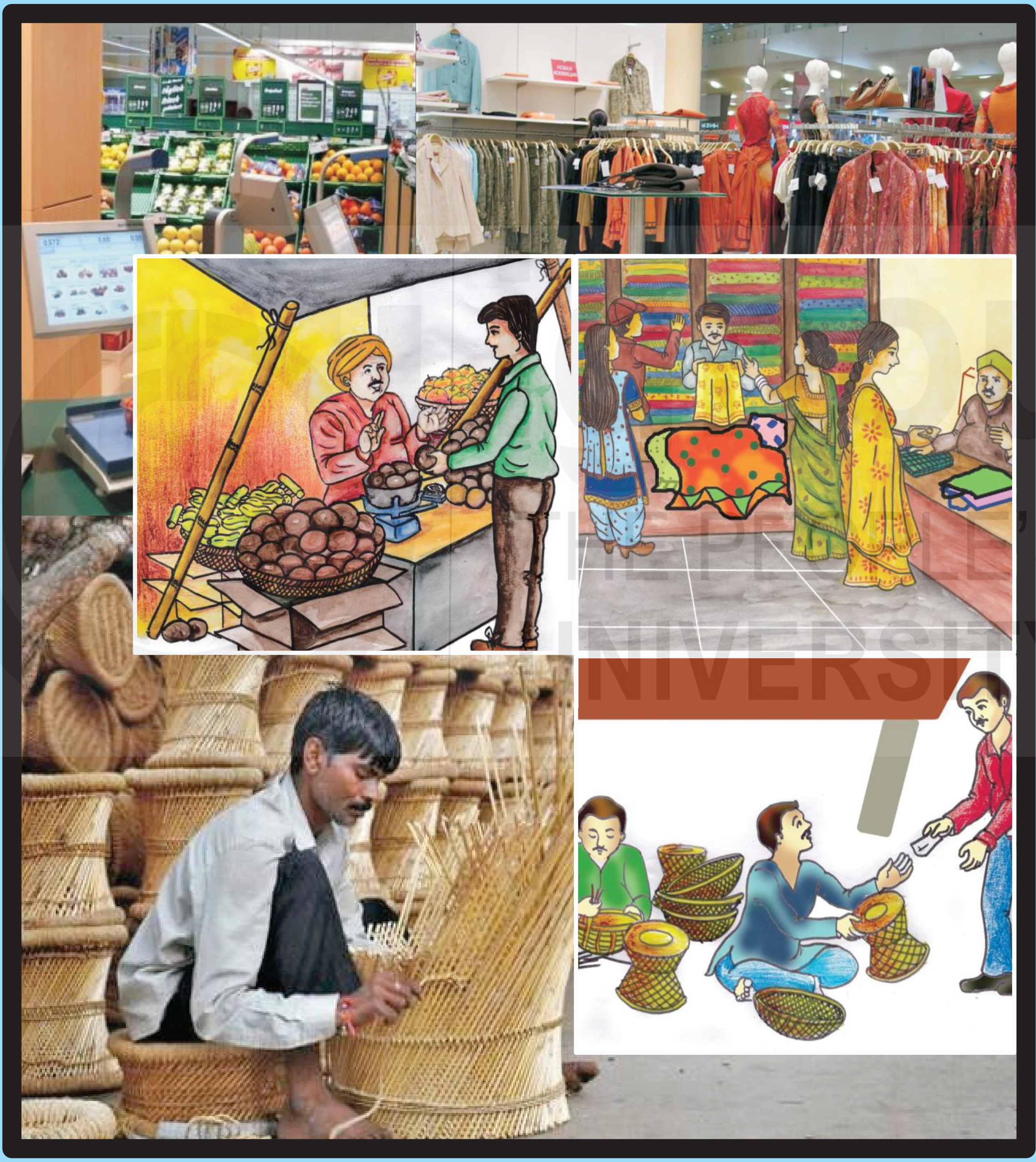
Indira Gandhi National Open University
School of Social Sciences

# BECC-105
# INTERMEDIATE MICROECONOMICS-I



BECC-105

# Intermediate Microeconomics-I

School of Social Sciences
Indira Gandhi National Open University

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## Course Contents

# COURSE INTRODUCTION: INTERMEDIATE MICROECONOMICS-I

Economics is a live subject and helps the economic agents in their decision making like: Which commodities to produce? How to produce? Which techniques to use? Which factors or resources to use, in which combinations to produce? What quantity of a commodity to produce? How consumers make purchasing decisions and how their choices are affected by changing prices and incomes? How firms decide how many workers to hire and how workers decide where to work and how much to work? Basically, it involves analysing the rational choice behaviour of individual economic agents.

This course on Intermediate microeconomics builds up on the Principles of Microeconomics course studied in Semester 1 and provides an analysis of how the economic theory developed can be directly applied to help economic agents in taking decisions pertaining to maximising utility, minimising cost, maximising profit, taking output or pricing decisions, etc. It achieves this through combining microeconomic theory with the application part using graphical analysis, algebra and calculus. In order to grasp this course well, a student is expected to have passed an introductory microeconomics course, and have some working knowledge of calculus (mostly derivatives), basic algebra and graphing skills.

The course structure is divided into 3 Blocks. Each block is further sub-divided in a number of Units. Each Unit in itself is self-contained and has organic linkages with all other Units. Throughout the course, in each unit, student will encounter a synchronized set of introductory theory, illustrative examples and check your progress exercises—designed to provide conceptual clarity to the student. In a way, this allows students to develop their abilities to evaluate, analyse and synthesise economic information.

The course broadly throws light on the consumer and producer theory, along with analysing the optimising behaviour of consumers and firms. It also takes up discussion on a competitive market structure and shed light on the efficiency status of it towards the end. The other market structures (like Monopoly, Monopolistic competition, Oligopoly) are discussed in Intermediate Microeconomics Part II that you will come across in Semester IV.

**Unit 1** introduces consumer's theory with a discussion on consumer's preferences and utility. **Unit 2** integrates the concepts discussed in Unit 1 and illustrates consumer's equilibrium using both the graphical and Lagrangian methods. Further it covers, how a commodity and income tax impact consumer's equilibrium, and how changes in commodity price and income influence consumer's equilibrium (using both Slutsky's and Hicksian's approach). **Unit 3** explains the concept of consumer's surplus, while **Unit 4** discusses choice under uncertainty and inter-temporal choice. Producer's theory begins with **Unit 5** which covers a discussion on the firm's production functions, the condition of producer's equilibrium and technological progress. The cost function, the condition of cost minimisation and factor demand functions have been covered in **Unit 6**. This lays the foundation for the study of behaviour of firms under different forms of market structure. For this semester we have restricted ourselves to the perfect competitive market in **Unit 7**. Finally, **Unit 8** of the course concludes with a discussion on the concept of efficiency or Pareto optimality, and analyses on the efficiency of a competitive market.

### Mathematical Symbols/ Notations used in the Course

| Symbol | Symbol Name/Meaning Attached | Usage |
|---|---|---|
| $\succeq$ | Weak Preference | A $\succeq$ B, alternative A is weakly preferred to B |
| $\sim$ | Indifference | A $\sim$ B, consumer is indifferent between alternative A and B |
| $\succ$ | Strict Preference | A $\succ$ B, alternative A is strictly preferred to B |
| $>; <; \geq; \leq$ | Greater than; Less than; Greater than or equal to; Less than or equal to, respectively | A $>$ B, A is greater than B; A $<$ B, A is less than B; A $\geq$ B, A is greater than or equal to B; A $\leq$ B, A is less than or equal to B, respectively |
| $[a, b]$ | Closed interval | [0, 2], an interval containing the set of real numbers that lie between numbers 0 and 2, including the numbers 0 and 2 themselves. |
| $(a, b)$ | Open interval | (0, 2), an interval containing the set of real numbers that lie between numbers 0 and 2, and not the numbers 0 and 2. |
| $\in$ | Element of | $t \in [0,1]$, $t$ is an element of the closed interval [0,1] |
| $\Delta$ | Delta/Change in | $\Delta$X, change in value of X |
| $dx$ | Differential | $dx$, represents an infinitely small change in the variable $x$ |
| $\Rightarrow$ | Implies | A$\Rightarrow$B, if A is true, then B is also true |
| F(.) or $f$ (.) | A function | F($x$) or $f$ ($x$), a function of $x$ |
| F′($x$) or $y'$ or $\frac{dy}{dx}$ | First-order derivative | Given $y$ = F($x$)—$y$ a function of $x$, F′($x$) or $y'$ or $\frac{dy}{dx}$ represent first-order derivative of function F($x$) with respect to $x$ |
| F″ (.)or $y''$ or $\frac{d^2y}{dx^2}$ | Second-order derivative | Given $y$ = F($x$)—$y$ a function of $x$, F″ (.)or $y''$ or $\frac{d^2y}{dx^2}$ represent second-order derivative of function F($x$) with respect to $x$ |
| $\lim_{\Delta x \to 0} f(x)$ | Limit | $\lim_{\Delta x \to 0} f(x)$, limit value of function $f$($x$) as $x$ approaches 0. |
| $\frac{\partial f(x,y)}{\partial x}$ or $\frac{\partial y}{\partial x}$ | Partial derivative | Given $y = f(x, y)$—$y$ a function of $x$ and $y$, $\frac{\partial f(x,y)}{\partial x}$ or $\frac{\partial y}{\partial x}$ represent partial derivative of function $f(x, y)$ with respect to $x$ |
| $ln$ | Natural log | $lnx$, log of $x$ to the base $e$, where $e$ = 2.718… |
| $\int_a^b f(x)dx$ | Definite Integral | $\int_a^b f(x)dx$ = F($b$) – F($a$), where F′($x$) = $f$($x$) |

**Some Greek Alphabets (and corresponding symbols) you may encounter:**

| Symbol | Greek Alphabet | Symbol | Greek Alphabet |
|---|---|---|---|
| $\alpha$ | Alpha | $\theta$ | Theta |
| $\beta$ | Beta | $\lambda$ | Lamda |
| $\gamma$ | Gamma | $\pi$ | Pi |
| $\delta$ | Delta | $\sigma$ | Sigma |
| $\varepsilon$ | Epsilon | $\chi$ | Chi |
| $\Psi$ | Psi | $\mu$ | Mu |
| $\rho$ | Rho | $\omega$ | Omega |

# Block 1
# Consumer Theory

# UNIT 1 PREFERENCES AND UTILITY

**Structure**



## 1.0 OBJECTIVES

After going through this unit, you should be able to:

- justify why a consumer prefers a particular bundle over the other available bundle;
- differentiate between weak and strict preferences;
- analyse the assumptions regarding well-behaved preferences;
- define marginal rate of substitution and underline importance of it for analysing consumers' behaviour;
- define properties of an indifference curve;
- establish link between a Utility function and Preference relation;
- construct an indifference curve from the given utility function;
- explain the link between the marginal utilities and the marginal rate of substitution; and
- figure out some examples of the utility functions and the underlying indifference curves.

## 1.1 INTRODUCTION

You were comprehensively introduced to the concepts of consumer behaviour through cardinal and ordinal approaches in Units 4 and 5 of your Introductory Microeconomics course of Semester 1 (BECC-101). The present unit makes use of that theory base and the mathematical techniques you came across in your Mathematical Methods in Economics course of Semester 1 (BECC-107) for examining the economic behaviour of the consumer. A consumer, be it an individual or a household, makes decision regarding which commodity or service to be purchased and in what quantities. What guides this decision making? Why does a consumer purchase a certain bundle of commodities? We know that he gets satisfaction or utility from consumption of commodities, but there also exist alternatives which can give him similar satisfaction. So why does our consumer choose a particular bundle of commodities over the other available bundles? What determines the preference behaviour? We shall discuss various aspects of preferences in Section 1.2.

A consumer derives utility or satisfaction from consumption of commodities. The extent of satisfaction can be estimated by a utility function, which gives an ordinal value to the consumption of a particular bundle of commodities. In the subsequent section, you will come across a concept like utility function, representing a specific preference relation. After deriving an expression for marginal utility, a relationship between the marginal rate of substitution and the marginal utilities will be established. The discussion will end with some examples of utility functions and the underlying indifference curves— both representing the same preference ordering.

## 1.2 CONSUMER'S PREFERENCES

A consumer makes decision about allocating his limited income among available goods in order to obtain maximum satisfaction or utility. For this, he/she chooses the *best* commodity bundle that he/she can *afford*. The affordability is determined by the budget constraint the consumer faces— which, in turn, depends upon his/her income and prices of the commodities; while the choice of the best bundle is guided by consumer's preferences.

Preferences are subjective individual tastes that permit a consumer to rank different bundles of goods on the basis of the utility they give to the consumer. Independent of consumer's income and goods' prices, preferences establish the relationships between the bundles of the commodities that a consumer faces. Assuming N commodities available for consumption, a commodity bundle is given by, $A = (x_1, x_2, x_3,\ldots,x_N)$, where $x_i$ with $i = 1,2,3,..N$ represents respective quantity of good 1,2,3, …,N. Given two bundles, A and B, if a consumer opts for bundle A when bundle B is available, then clearly bundle A is preferred to bundle B by this consumer. Please note — preferences establish relationship between bundles of commodities and not among individual commodities.

The consumer may prefer bundle A strictly over bundle B, or he/she might regard bundle A at least as good as bundle B (that is, not inferior to B). There may be a possibility that consumer fails to prefer bundle A over B—he/she may find them as good as one another. We are going to use certain symbols to denote various notions of preferences.

### 1.2.1 Weak and Strict Preference

**Weak Preference :** When a consumer considers bundle A to be at least as good as bundle B, we say he weekly prefers bundle A over B. Symbolically, this is denoted by: $A \succeq B$

**Indifference:** When both bundles A and B are regarded as good as one another. That is:

$$A \succeq B \text{ and } B \succeq A$$

We say that consumer is indifferent between bundle A and B, denoted by: $A \sim B$

**Strict Preference:** When bundle A is regarded as superior to B, then the relationship is that of strict preference, represented by: $A \succ B$

So, if $A \succeq B$ and neither $A \sim B$ nor $B \succeq A$, then we have $A \succ B$, or in words, "A is strictly preferred over B".

### 1.2.2 Assumptions about Preferences

Here, we are specifying certain assumptions about preferences. These assumptions help us in developing the theory of consumers' choice in a systematic manner. Preferences exhibit three important properties. They are:

- Completeness
- Reflexivity
- Transitivity

**Completeness**

By completeness it simply means, available bundle options can be compared. That is, for bundles A and B, either $A \succeq B$, or $B \succeq A$, or both (i.e., $A \sim B$). This means that it is always possible for a consumer to say whether or not he/she would prefer one bundle to another. There is no gap in the choice set, the consumer can make unambiguous choices on the assumption that he/she does not suffer from lack of information about the bundles he/she is asked to make a choice from.

**Reflexivity**

For any bundle A, $A \succeq A$. That is to say, any bundle A is at least as good as itself.

**Transitivity**

If bundle A is at least as good as bundle B, bundle B is at least as good as bundle C, then bundle A is at least as good as bundle C. Symbolically, If A $\succeq$ B and B $\succeq$ C

Then, A $\succeq$ C

If this condition is not satisfied the consumers' behaviour may suffer from irrational preference circularity, he may end up saying A $\succ$ B and B $\succ$ C but C $\succ$ A!

Given this background we can move into depiction of preferences with help of indifference curves.

*Please Note*: You were comprehensibly introduced to the concepts related to consumer theory in your Introductory Microeconomics course of Semester 1. We briefly present some concepts and theory here.

### 1.2.3 The Indifference Curve

Indifference curve is a locus of all the combinations of two goods that provide a constant level of satisfaction or utility to a consumer. Consider Fig. 1.1 below. Here combination bundles represented by point A, B, C give the consumer same level of utility, so that A ~ B ~ C.

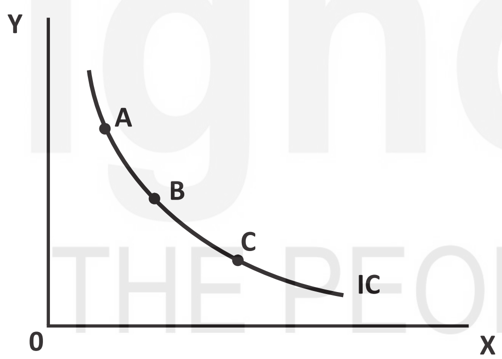


**Fig. 1.1: An Indifference Curve**

In Fig. 1.2 (a) and (b), we represent the set of bundles (given by the shaded region) weakly preferred to bundle A, and the set of bundles strictly preferred to bundle A, respectively. As you may notice,

In part (a) indifference curve forms the part of the set (the shaded area) of the bundles weakly preferred to bundle A. For instance, consumer will be indifferent between bundle B which belongs to this set and bundle A, as both are a part of the indifference curve, whereas bundle C which also is a part of this set, will be strictly preferred to bundle A or B, as it contains more of both the goods (X and Y) than is contained in bundles A or B.

In part (b) indifference curve is not included in the set (the shaded area) of bundles strictly preferred to bundle A, to show which we have constructed a dotted curve. Here, consumer is indifferent between bundle A and B, the reason bundle B does not forms the part of the set of bundles strictly preferred to bundle A. Bundle C on the other hand is strictly preferred to bundle A and thus forms the part of this set.

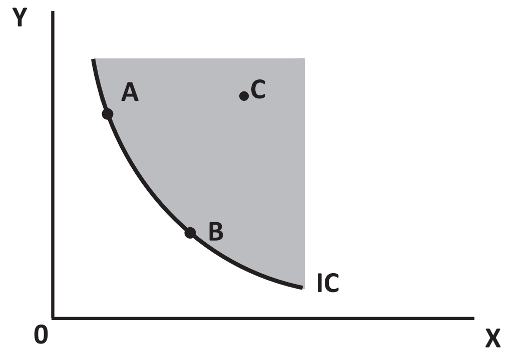


Fig. 1.2 (a): Set of Bundles Weakly preferred to A

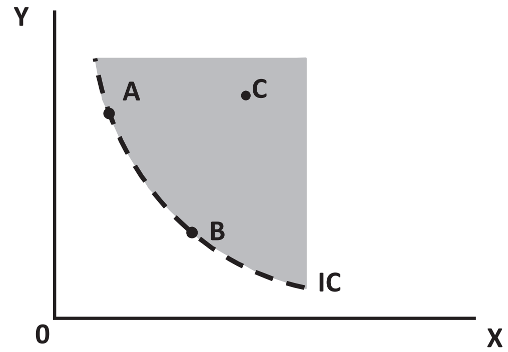


Fig. 1.2 (b): Set of Bundles Strictly preferred to A

**Indifference Map**

Entire set of indifference curves reflecting tastes and preference of a consumer in the form of different utility levels for the two goods is referred to as the indifference map. In Fig. 1.3, $IC_1$, $IC_2$, $IC_3$ represent such a set.

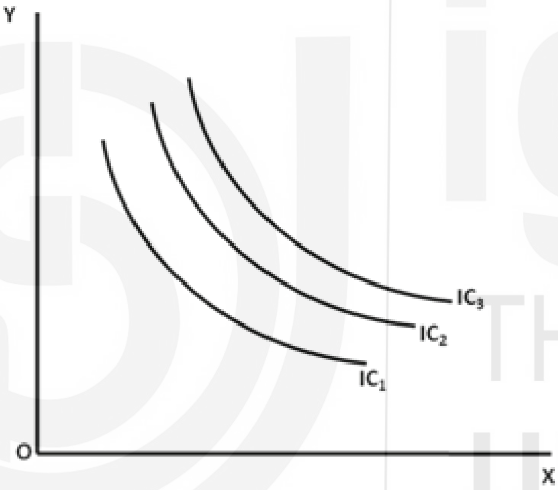


Fig. 1.3: Indifference Map

## 1.2.4 Well-behaved Preferences

In addition to the assumptions of reflexivity, transitivity and completeness, we usually make two further assumptions about consumers' well-behaved preferences. A preference relation is said to be "well-behaved" if it is monotonic and convex.

i) **Monotonicity:** Monotonic preference means that a rational consumer always prefers more of a commodity as it offers him a higher level of satisfaction. Monotone preferences essentially say that "more" is preferred to "less".

A consumer's preferences are said to be *weakly monotonic* if, given a consumption bundle A ($X_1$, $Y_1$), the agent prefers all consumption bundles B ($X_2$, $Y_2$), that have more of every good, *i.e.*, $X_2 > X_1$ and $Y_2 > Y_1$ ($a\ two\ commodity\ framework$) implies B $\succ$ A. A consumer's preferences are said to be *strongly monotonic* if, given a consumption bundle A ($X_1$, $Y_1$), the agent prefers all consumption bundles B ($X_2$, $Y_2$)

that have more of at least one good, and not less in any other good, *i.e.*, either $X_2 > X_1$ and $Y_2 = Y_1$ *or* $X_2 = X_1$, $Y_2 > Y_1$ (*a two commodity framework*) imply $B \succ A$. This assumption simply says— "the more, the better", so that a consumer prefers consuming more of a good to consuming less of it. That is, considering two bundles A and B, with bundle B having at least as much of all the goods as bundle A, and more of one, then $B \succ A$. This implies that indifference curve has a negative slope. You may observe this yourself (just think of a positively sloped indifference curve representing bundles of commodities with more of both the goods).

Note that, assumption of monotonicity cannot determine the order of two bundles if one bundle has higher quantity of some commodities and smaller quantity of others.

ii) **Convexity:** The assumption of convexity says that weighted average of commodity bundles is preferred to extreme bundles. Consider two commodity bundles A and B on the indifference curve in Fig. 1.4. Weighted average of these bundles will be given by any point (depending upon the weight given to extreme bundles) on the line connecting both of them. As you may notice, the weighted average points lie on the area representing bundles which are preferred to the indifference curve on which extreme bundles (A and B) lies. This explains why consumer prefers weighted average to extremes. The assumption of convexity implies consumer's preference is subject to diminishing marginal utility.

Symbolically, bundle C, where C is given by $tA + (1 - t)B$ or $[tX_1 + (1 - t)X_2, tY_1 + (1 - t)Y_2]$ with $t \in [0,1]$ will be preferred to bundle A $(X_1, Y_1)$ or B $(X_2, Y_2)$.

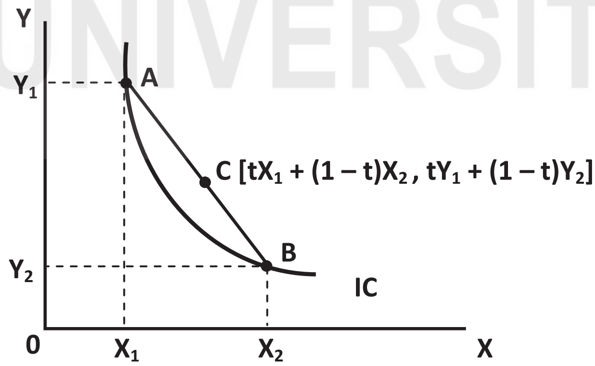


**Fig. 1.4: Convex preferences**

### 1.2.5 Marginal Rate of Substitution (MRS)

MRS is the rate at which consumer is willing to trade-off consumption of one commodity for consumption of the other, without affecting his level of satisfaction. Consider Fig. 1.5, suppose consumer is initially consuming bundle A. If he increases consumption of good Y by ΔY and reduces that of good X by ΔX, then marginal rate of substitution between good X and Y ($MRS_{XY}$) will be given by $\frac{\Delta Y}{\Delta X}$.

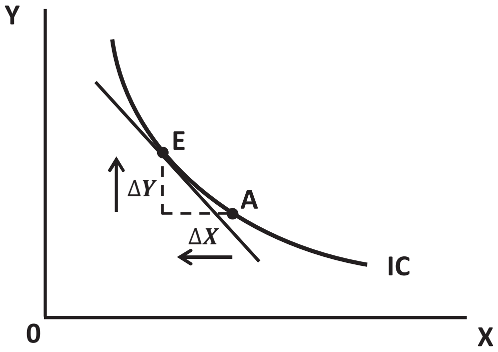


**Fig. 1.5: Marginal Rate of Substitution**

If we allow ΔX and ΔY to be very small, the ratio $\frac{\Delta Y}{\Delta X}$ will approach slope of IC at point E, which is then given by the slope of the tangent (i.e. $\frac{dY}{dX}$) to the point E. Thus, with infinitesimal small ΔX and ΔY, $MRS_{XY}$ represent slope of indifference curve at a point. Mathematically,

$$MRS_{XY} = -\lim_{\Delta X \to 0} \frac{\Delta Y}{\Delta X} = -\frac{dY}{dX}$$

That is, $MRS_{XY}$ represents the limiting value of the ratio $\frac{\Delta Y}{\Delta X}$ as the denominator approaches zero. As you may notice, we have inserted a negative sign in order to get $MRS_{XY}$ as a positive quantity. This is done because indifference curve is negatively sloped with ratio $\frac{\Delta Y}{\Delta X}$ already possessing a negative sign.

### 1.2.6 Properties of Indifference Curves

1) Indifference curves are negatively sloped.
2) Indifference curves describing two distinct levels of utility cannot intersect or cross each other. This is a result of the transitivity assumption.

   **Proof:** Consider Fig. 1.6, where we have two intersecting *alleged* ICs, $IC_1$ and $IC_2$. Consider points A and B, they lie on $IC_2$ therefore, A ~ B.

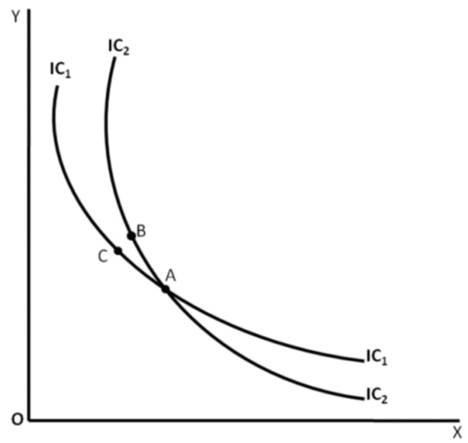


**Fig. 1.6**

But A and C lie on $IC_1$, therefore $A \sim C$. However, B lies to North-east of C, therefore $B \succ C$. Hence we have a contradiction: $A \sim C$ and $A \sim B \Rightarrow B \sim C$ (from transitivity assumption), but $B \succ C$. Both these statements cannot hold together. Therefore, $IC_1$ and $IC_2$ cannot intersect.

3) An indifference curve is usually convex to the origin. That is, slope diminishes as consumer substitute commodity X for commodity Y. This results from the fact that $MRS_{XY}$ falls as we move down along an indifference curve. As more and more units of commodity X are consumed, consumer is willing to give up lesser and lesser units of commodity Y. The reason for this is that marginal utility from consumption of a good falls as more and more units of it are consumed. So with increase in consumption of X, marginal utility of it falls, while marginal utility of commodity Y rises, resulting in consumer's willingness to give up fewer units of Y for X.

**Check Your Progress 1**

1) Differentiate between strict and weak preference.

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2) Explain the three important properties of preferences with examples.

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3) Explain the properties of indifference curves?

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4) What is the notion of:

i) Convexity

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ii) Monotonicity

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## 1.3 UTILITY

Synonymous with "satisfaction", "well-being", "pleasure", etc., the concept of utility has evolved over time in economic literature. Alfred Marshall considered utility to be real, measurable, *i.e.* of cardinal scale. He had assumed that utility accruing to a consumer from consumption of a unit of a commodity could be measured in terms of a cardinal number having the unit called 'util'. But the problem in this approach was that of unavailability of an appropriate measurement index giving that cardinal number. For instance — whether 1 util for consumer A is equivalent to 1 util for consumer B? or does increasing utility from 1 to 2 utils indicate doubling of the utility attained? — are some of the issues left unsolved by the cardinal approach. Ordinal approach by J.R. Hicks, being unit-free, was able to overcome these problems, by way of 'ranking' and 'preference-ordering' bundles. According to this approach, one could 'order' different bundles as 'better', 'worse', or 'as good as', but saying nothing about the strength of the preferences.

### 1.3.1 Utility Function and Preferences

A utility function (U) defines the level of utility attained by a consumer as a function of the amount of commodities consumed by him. The function takes the following form:

$$U_1 = U(X, Y)$$

where X and Y represent the quantities of commodities consumed by the consumer, and $U_1$ is the utility level (a number) obtained from consuming this commodity bundle.

The utility function can be derived from preferences, or in other words, preferences can be represented by utility functions. The function U assigns values to different bundles that exactly reflect consumer's preferences, that is,

$$U(A) \succeq U(B) \text{ if and only if } A \succeq B$$

Now, here A and B represent bundles of commodities X and Y.

As you may notice above, the preference relation among the bundles is preserved by the utility function. That is, if a consumer weakly prefers bundle A to B, then utility obtained from consumption of bundle A will not be less than that obtained from consumption of bundle B. For utility function to represent preference ordering, preference relation must be complete, transitive, reflexive, and continuous. By continuity it means that the preference relation has "no jumps", that is, if bundle A is strictly preferred to B, then bundles "close to" A are also preferred to B.

**Remember:** Values given by utility functions only have ordinal meaning, that is, only the ordering of the numbers matter and not the cardinality (the difference between the numbers). It simply means a utility function U representing preference for bundles A, B and C by assigning utility numbers as U(A) = 1, U(B) = 2 and U(C) = 3, will represent the same preferences if utility numbers would have been U(A) = 1, U(B) = 1.5 and U(C) = 2. This implies that same preferences could be represented by many utility functions.

***Relation between two Utility functions representing same Preferences:*** Considering two utility functions— U and V. They both will represent same preference if and only if, there exists a strictly increasing function F such that

$$V = F(U), \text{ such that } F'(U) > 0$$

For instance, if we define V= U + C, where C is any constant, then V(A) $\succeq$ V(B) if and only if U(A) $\succeq$ U(B), if and only if A $\succeq$ B. Function V is any transformation of function U that leaves the preference ordering representation intact. Such transformations are called *monotonic* transformation. Thus, if a utility function represents a consumer's preferences, then a monotonic transformation of that utility function will result in another utility function representing the same preferences.

### 1.3.2 Utility Function and Indifference Curve

We just discussed— a utility function $U_1 = U(X, Y)$ represents the preferences of a consumer. Also, we know that an Indifference curve links bundles which yield the same level of utility. Thus, an indifference curve can be graphically represented by a function of quantities of two commodities yielding same level of utility, or in other words, a representation of a utility function with a given level of utility value. We can obtain such a function by setting $U_1$ on the left-hand side of the utility function equal to some constant value, like 10, 12, etc. and then express Y as a function of X. Let us consider an example:

Let our utility function be given by, U(X, Y) = XY, setting it equal to a constant number 'K', we get,

$$XY = K$$

It can be solved for Y, such that $Y = \frac{K}{X}$

Now, for different levels of K, *i.e.* 1, 2, 3, we can obtain a set of indifference curves, which constitute our indifference map in Fig. 1.7.

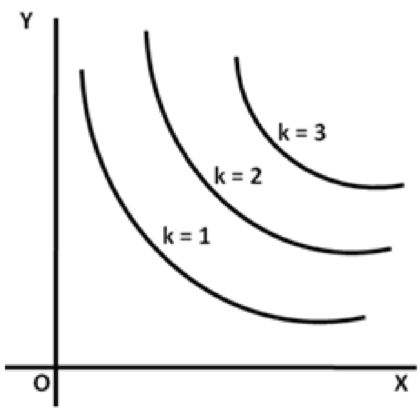


**Fig. 1.7**

### 1.3.3 Marginal Utility (MU)

Marginal utility is the change in total utility resulting from a small change in the quantity of one of the commodities consumed holding constant the quantity of the other commodity. For a given utility function U(X,Y), marginal utility with respect to commodity X will be given by,

$$MU_X = \lim_{\Delta X \to 0} \frac{\Delta U}{\Delta X} = \lim_{\Delta X \to 0} \frac{U(X+\Delta X, Y) - U(X,Y)}{\Delta X}$$

This implies, change in total utility resulting from change in consumption of commodity X, $\Delta U = MU_X \, \Delta X$

Similarly, marginal utility with respect to commodity Y will be given by,

$$MU_Y = \lim_{\Delta Y \to 0} \frac{\Delta U}{\Delta Y} = \lim_{\Delta Y \to 0} \frac{U(X, Y+\Delta Y) - U(X,Y)}{\Delta Y}$$

With change in total utility resulting from change in consumption of commodity Y,

$$\Delta U = MU_Y \, \Delta Y$$

When $\Delta X$ and $\Delta Y$ approaches zero, or in other words, when change in the commodities become infinite small, then the marginal utilities are derived as a partial derivative of the utility function with respect to X in case of $MU_X$ and with respect to Y in case of $MU_Y$, that is

$$MU_X = \frac{\partial U(X,Y)}{\partial X} \text{ and } MU_Y = \frac{\partial U(X,Y)}{\partial Y}$$

**Remember:** The magnitudes of MU will depend upon the specific utility function reflecting consumer's preference behaviour, that is, their own magnitude will have no particular significance. Despite having no behavioural content of its own, the MU can help in calculating something with behavioural content. This is marginal rate of substitution (MRS).

### 1.3.4 Relationship between MU and MRS

Indifference curve is a locus of those bundles of X and Y which give same level of utility or satisfaction to our consumer. Therefore, it can be represented in a functional form:

$$U(X, Y) = U_1$$

where $U_1$ represents a given utility level.

Differentiating this function totally, we get

$$\frac{\partial U(X,Y)}{\partial X} . dX + \frac{\partial U(X,Y)}{\partial Y} dY = dU_1$$

As utility remains constant along an IC, therefore $dU_1$ will be 0.

$$MU_X \, dX + MU_Y \, dY = 0$$

$$[\text{where } MU_X = \frac{\partial U(X,Y)}{\partial X} \text{ and } MU_Y = \frac{\partial U(X,Y)}{\partial Y}]$$

or $$MU_X \, dX = - MU_Y \, dY$$

$$\text{or} \qquad \frac{MU_X}{MU_Y} = -\frac{dY}{dX} = MRS_{XY}$$

This makes possible another interpretation of the MRS. Marginal Rate of Substitution is ratio of marginal utilities of the two goods.

### 1.3.5 Utility Functions and Underlying Indifference Curves: Some Examples

Let us now consider some examples of utility functions and the underlying indifference curves:

**Perfect Substitutes**

Commodities which are Perfect substitute to each other are said to have a constant rate of trade-off ($MRS_{XY}$) between them. Utility function in this case takes the form:

$$U(X, Y) = aX + bY, \text{ where } a, b > 0$$

where 'a' units of X can be substituted for by 'b' units of Y. Now, $MU_X = \frac{\partial U(X,Y)}{\partial X} = a$ and $MU_Y = \frac{\partial U(X,Y)}{\partial Y} = b$

slope is given by

$MRS_{XY} = \frac{MU_X}{MU_Y} \Longrightarrow MRS_{XY} = \frac{a}{b}$ (which is a constant, independent of X and Y)

Underlying indifference curve will be linear with a constant slope, $-\frac{a}{b}$ (refer Fig.1.8)

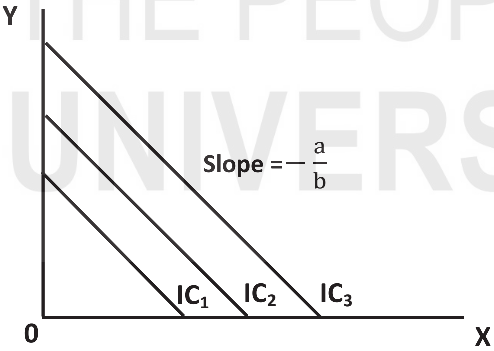


**Fig.1.8: Indifference Curves when goods are Perfect Substitutes**

**Perfect Compliments**

When commodities are perfect compliments, they are consumed in fixed proportion (not necessarily 1:1). Utility function takes the form:

$$U(X, Y) = \min (aX, bY), \qquad \text{where } a, b > 0$$

Indifference curves will have L-shape with kinks at points A, B, C, where aX = bY (refer Fig. 1.9). $MRS_{XY}$ equals 0 along the vertical part of the curve, and infinity along the horizontal part of the curve, whereas, along the kinks, $MRS_{XY}$ is not defined (as you may notice no unique tangent can be drawn at the kinks).

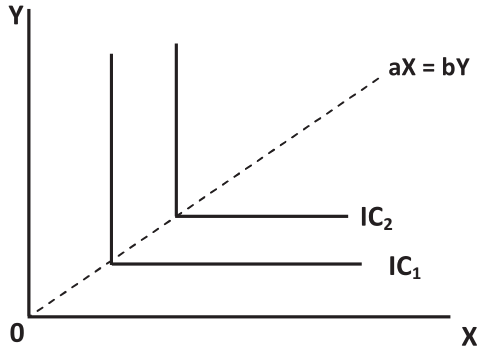


Fig. 1.9: Indifference Curves when goods are Perfect Complements

## Cobb-Douglas

Cobb-Douglas utility function is used very often in economic analysis. It is specified as:

$$U(X, Y) = X^c Y^d$$

where positive numbers c, d, describes relative importance of the commodities.

$$MRS_{XY} = \frac{MU_X}{MU_Y} \Longrightarrow MRS_{XY} = \frac{cX^{c-1}Y^d}{dX^cY^{d-1}} = \frac{c}{d}\frac{Y}{X}$$

In the utility function $U(X, Y) = X^c Y^d$, let the utility level be K, *i.e*. we have $K = X^c Y^d \Longrightarrow Y = K^{1/d} X^{-c/d}$. Substituting value of Y in expression for $MRS_{XY}$, we get

$$MRS_{XY} = \frac{c}{d} K^{1/d} X^{-(c+d)/d}$$

Here $MRS_{XY}$ decreases $(\frac{\partial MRS_{XY}}{\partial X} < 0)$ with increase in X, that is, $MRS_{XY}$ is diminishing as we move down the indifference curve, resulting in convex-shaped ICs. General shapes for Cobb-Douglas indifference curves are indicated in Fig. 1.10a and 1.10b. Note that shapes vary as per relative magnitudes of c and d.

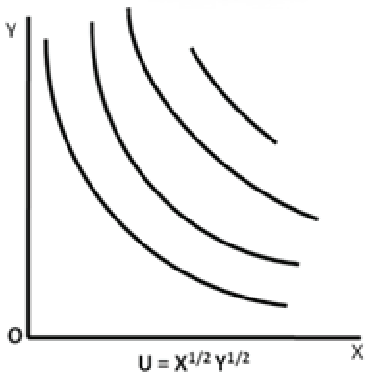


Fig. 1.10 (a)

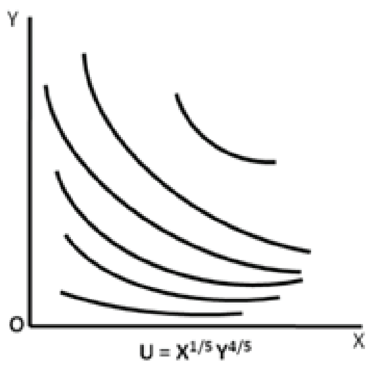


Fig. 1.10 (b)

One reason for popularity of C-D indifference curves is that these are well behaved indifference curves. The formula $U = X^c Y^d$, where c + d = 1 is the simplest algebraic expression that generates well-behaved ICs. Their monotonic transformation will also give same well behaved set of ICs. For example: if we have $U(X, Y) = X^c Y^d$, then a utility function given by V = ln [U (X, Y)] $\Longrightarrow$ V= ln $(X^c Y^d)$ = c lnX + d lnY gives the same preference relation or set of ICs. We can generate similar ICs through another transformation.

We have $\qquad U(X, Y) = X^c Y^d$

Raising utility function to the power $\frac{1}{c+d}$, we get

$$[U(X, Y)]^{1/(c+d)} = X^{c/(c+d)}.\ Y^{d/(c+d)}$$

Assuming $a = \frac{c}{c+d}$, then $1 - a = 1 - \frac{c}{c+d} = \frac{d}{c+d}$

Now, we can rewrite the above function as

$$U(X, Y) = X^a Y^{1-a}$$

Thus, a monotonic transformation of a Cobb-Douglas utility function will be Cobb-Douglas function whose exponents add up to unity.

**Quasi-linear**

Quasi-linear utility function is a function which is linear in one commodity (let say Y) and non-linear in the other (here X), that reason it is called quasi-linear. The function is given by

$$U(X, Y) = f(X) + Y\ ;\ f'(X) > 0,\ f''(X) < 0$$

Now, $MRS_{XY}$ will be given by, $MRS_{XY} = \frac{MU_X}{MU_Y} \Longrightarrow MRS_{XY} = f'(X)$.

Note here that $MRS_{XY}$ only depends upon X and not on Y; hence ICs are parallel shifts of each other as shown in Fig. 1.11.

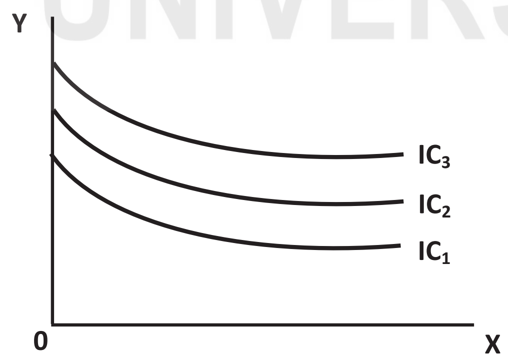


**Fig. 1.11 : Quasi-linear (non-linear in X here)**

An example of a quasi-linear function could be, U(X, Y) = *ln* X + Y, where '*ln*' represents 'natural log— log to the base e'. Here, $MRS_{XY} = \frac{1}{X}$.

**Check Your Progress 2**

1) Distinguish between cardinal and ordinal utility.

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2) A preference relation, in order to be represented by a utility function must satisfy what all properties?

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3) Derive the relation between the Marginal rate of substitution and the marginal utilities.

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## 1.4 LET US SUM UP

The unit has described the theory of consumer's preferences. We had some detailed discussion about the concepts related to the preferences. We distinguished between weak and strict preferences, defined the notion of indifference curves and map, discussed assumptions about preferences, and particularly about well-behaved preferences. All this has set forth the base for analysing a particular preference behaviour of a consumer.

After some brief introduction of the concept of utility, we came across the function that assigns a numerical value corresponding to the level of utility obtained from consumption of commodities bundles — called the utility function. We established the link between the utility function and the indifference curve, as both represented the same preference ordering. Subsequently, the concept of marginal utility was explained, and further to this, relationship between marginal rate of substitution and marginal utilities was derived. We concluded the unit by presenting some examples of utility functions along with the underlying indifference curves, with both representing the same preference relation.

## 1.5 REFERENCES

1) Varian, HR, (1999). *Intermediate Microeconomics: A Modern Approach* 5th Edition.

2) Newman , Peter, (1965). *Theory of Exchange*.

## 1.6 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

**Check Your Progress 1**

1) Strict preference: A is at least as good as B and B is not at least as good as A. Weak preference: A is at least as good as B but A need not be superior to B.

2) Reflexivity, for all A, A $\succeq$ A.

   Completeness, for all A, B either A $\succeq$ B or B $\succeq$ A;

   Transitive, for all A, B and C, if A $\succeq$ B and B $\succeq$ C then A $\succeq$ C

3) A normal well-behaved indifference curve is:

   i) Higher Indifference curve indicate higher level of utility;

   ii) Monotonically sloping downwards to the right;

   iii) Convex to the origin; and

   iv) Two indifference curves do not touch or intersect.

4) Refer Sub-section 1.2.4 and answer.

**Check Your Progress 2**

1) Cardinal Utility: Utility is exactly measurable, Ordinal utility: Utility is not exactly measurable but ordered so that one can compare utilities from two bundles and say which one is giving higher satisfaction.

2) For utility function to represent preference ordering, preference relation must be complete, transitive, reflexive, and continuous.

3) Refer Sub-section 1.3.4 and answer.

# UNIT 2 CONSUMER'S EQUILIBRIUM

**Structure**



## 2.0 OBJECTIVES

After going through this unit, you will be able to :

- state the concept of consumer's equilibrium;
- find out the optimal consumption bundle of the consumer using the Lagrangian method;
- describe the indirect utility function, the expenditure function, the equi-marginal principle, marginal rate of substitution and the marginal utility of income;
- critically analyse the consumer's equilibrium in the real world phenomena;
- discuss the impact of economic policy tool such as taxes on the consumer's equilibrium;
- use Slutsky's and Hicksian approach to decompose the price effect into income effect and substitution effect; and

- explain the extent of substitution and the income effect of a price change in case of perfect substitutes, perfect compliments and quasi-linear preferences.

## 2.1 INTRODUCTION

How does a consumer with a limited income decide which goods and services to buy, constitutes the essence of consumer behaviour. In this context it becomes relevant to know how consumers allocate their incomes across goods and services. This, in turn, enables us to know how changes in income and prices affect the demand for goods and services. To answer these questions you need to be exposed to the consumer preferences, budget constraints and consumer choices. Hence all these three concepts have been explained in Unit 1.

We have learned in the previous unit that consumers as an economic agent have preferences among the various goods and services available to them. Further they face budget constraints which put limits on what they can buy. In this unit we shall explain the consumer's optimisation (consumer's equilibrium) i.e. how the consumers decide which combination of goods and services to buy, given their income and prices so as to maximise their satisfaction, pre-supposing that consumers are rational and well informed. For this purpose we shall illustrate the application of Lagrangian method. The impact of income tax vs commodity tax on consumer's equilibrium will be examined in the subsequent section. How the changes in price and income influence the consumer's equilibrium have also been elaborated with the help of Slutsky's approach and Hicksian's approach.

Let us begin with explaining the concept of consumer's equilibrium.

## 2.2 THE CONCEPT OF CONSUMER'S EQUILIBRIUM

This section focuses on what are thought to be fairly common factors that influence consumer behaviour. Two factors are emphasised. First is the objective ability of the consumer to acquire goods, which is determined by income and by the prices of the goods. Second are the subjective attitudes, or tastes, of the consumer concerning the relative desirability of various combinations of goods and services. Indifference curves represent the consumer's subjective attitude towards various market baskets whereas the budget line shows what market baskets the consumer can afford. Putting the two pieces of apparatus together, we can determine what market basket the consumer will actually choose.

***The consumer is said to be in equilibrium when he maximises his satisfaction, given his money income and the prices of the commodities he consumes.***

The following assumptions are made in order to explain how a consumer reaches the equilibrium position:

1) The consumer acts rationally, that is, he is guided by the objective to maximise his satisfaction.

2) The prices of the goods are given and remain unchanged.

3) He has a given amount of money income to spend on the goods. There are no savings.

4) The scale of preferences of the consumer for various combinations of the two goods, X and Y, are represented by an indifference map, which is defined by his utility function. This scale of preferences remains the same throughout the analysis.

5) Each of the good is an economic 'good' and is assumed to be completely divisible.

The two conditions need to be fulfilled for a consumer to be in equilibrium:

**Necessary Condition**

The budget line must be tangent to the indifference curve i.e.

$$MRS_{X,Y} = \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

Marginal rate of substitution (MRS) is a measure of the consumer's subjective marginal benefit. On the other hand, the price ratio is effectively a measure of marginal cost. At equilibrium, marginal benefit is equal to marginal cost. This is necessary but not a sufficient condition for equilibrium.

**Sufficient Condition**

At the point of tangency, the indifference curve must be convex to the origin, that is, $MRS_{xy}$ must be diminishing at the point of tangency. Graphically, given the indifference map of the consumer and his budget line, the equilibrium is defined by the point of tangency of the budget line with the highest possible indifference curve as is represented by point e* in Fig. 2.1. The consumer will reach equilibrium at point e* (*i.e.* he will purchase OX* units of good X and OY* units of good Y) where the budget line RS is tangent to the highest possible indifference curve $IC_2$.

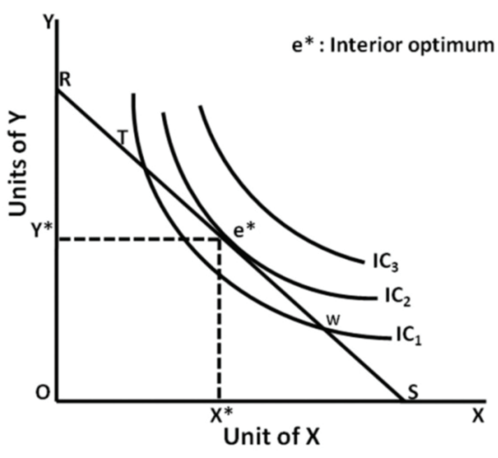


**Fig. 2.1**

i) The consumer would not like to choose a combination of X and Y represented by point T or W (although they lie on the budget line RS), because he will be on a lower indifference curve $IC_1$ and would thus be getting less satisfaction vis-à-vis point e*, which is on the same budget line RS but on a higher indifference curve, $IC_2$.

ii) The consumer cannot move to indifference curve $IC_3$, as this is beyond his means (money income) given by the budget line.

iii) Even on the indifference curve $IC_2$, all other points, except e*, are beyond his means.

Therefore, the optimal consumption position is where the indifference curve is tangent to the budget line, given by e* = (X*, Y*), where

Slope of indifference curve = Slope of budget line

$$MRS_{xy} = \frac{P_X}{P_Y}$$

## 2.3 CONSUMER'S EQUILIBRIUM WITH THE METHOD OF LAGRANGIAN MULTIPLIER

The method of Lagrangian multiplier is a technique that can be used to maximise or minimise a function subject to one or more constraints. The aim of the consumer is to purchase some quantities of the two goods which maximise his total utility *i.e.*

$$\text{Maximise} \quad U(X, Y) \tag{1}$$

subject to the constraint that all income is spent on the two goods:

$$P_x X + P_y Y = M \tag{2}$$

Here, U(X, Y) is the utility function, X and Y the quantities of the two goods purchased, $P_x$ and $P_y$ the prices of the goods, and M is the income.

This is a constrained optimisation problem which can be solved for optimal quantities of the two goods (X* and Y*) through a mathematical technique called Lagrangian method which is explained below:

1) **Stating the Problem:** First, we write the Lagrangian for the problem. The Lagrangian is the function to be maximised or minimised (here, utility is being maximised), plus a variable denoted by 'λ' times the constraint (here, the consumer's budget constraint), λ > 0.

   **The Lagrangian funct**ion is then given by

$$\mathcal{L} = U(X, Y) - \lambda (P_x X + P_y Y - M) \tag{3}$$

   Note that we have written the budget constraint as

$$P_x X + P_y Y - M = 0$$

   *i.e.*, entire income is exhausted in consumption of X and Y. We t**h**en multiplied it by λ and subtracted it from U function.

2) **Differentiating the Lagrangian:** If we choose values of X and Y that satisfy the budget constraint, then the second term in Equation (3) will be zero. Maximising will therefore be equivalent to maximising U(X, Y). By differentiating $\mathcal{L}$ with respect to X, Y and λ and then equating the derivatives to zero, we obtain the necessary conditions for a maximum. You should note that these conditions are necessary for an 'interior' solution in which the consumer consumes positive amounts of both goods. The solution, however, could be a "corner" solution in which all of one good and none of the other is consumed. But the corner solution cannot be solved by the Lagrangian technique. The implicit assumption of Lagrangian technique is that the solution is interior.

The resulting equations from the first order conditions are:

$$\left.\begin{aligned}\frac{\partial \mathcal{L}}{\partial X} &= \frac{\partial U(X,Y)}{\partial X} - \lambda P_X = 0 \Longrightarrow MU_X = \lambda P_X \\ \frac{\partial \mathcal{L}}{\partial Y} &= \frac{\partial U(X,Y)}{\partial Y} - \lambda P_Y = 0 \Longrightarrow MU_Y = \lambda P_Y \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= P_X X + P_Y Y - M = 0 \Longrightarrow P_X X + P_Y Y = M\end{aligned}\right\} \quad (4)$$

Here as before, MU stands for marginal utility. That is, $MU_X = \frac{\partial U\,(X,Y)}{\partial X}$, the incremental change in utility from a very small increase in the consumption of good X.

3) **Solving the Resulting Equations:** The three Equations in (4) can be rewritten as

$$\left.\begin{aligned}MU_X &= \lambda P_X \\ MU_Y &= \lambda P_Y \\ P_X X + P_y Y &= M\end{aligned}\right\} \quad (5)$$

Now we can solve these three equations for the three unknowns (X, Y and λ) in terms of the parameters $P_X$, $P_Y$, M. The resulting values of X and Y are the solution to the consumer's optimisation problem. Thus the utility maximising quantities are $X^*(P_X, P_Y, M)$ *and* $Y^*(P_X, P_Y, M)$. The functions $X^*$and $Y^*$, called the ordinary or uncompensated or Walrasian or Marshallian demand functions, are the functions of own price ($P_x$), cross price ($P_Y$) and income (M) of the consumer.

## 2.3.1 The Equi-Marginal Principle

The third equation in the Equation set (5) is the consumer's budget constraint with which we started. The first two equations in (5) tell us that each good will be consumed up to the point at which the marginal utility from consumption is a multiple (λ) of the price of the good. To see the implication of this, we combine these first two equations to obtain the equi-marginal principle:

$$\lambda = \frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \quad (6)$$

In other words, the marginal utility of each good divided by its price is the same. To optimise, the consumer must get the same utility from the last rupee spent by consuming either X or Y. If this is not the case, consuming more of one good and less of the other would increase utility.

To characterise the individual's optimum in more detail, we can rewrite the information in (6) to obtain

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y} \quad (7)$$

In other words, the ratio of the marginal utilities is equal to the ratio of the prices.

**Disequilibrium Cases**

- If $MRS_{xy} > \frac{P_X}{P_Y} \Longrightarrow \frac{MU_X}{MU_Y} > \frac{P_X}{P_Y} \Longrightarrow \frac{MU_X}{P_X} > \frac{MU_Y}{P_Y}$, the consumer must increase the consumption of good X (as he obtains greater per rupee utility from consumption of good X) till the equality between the $MRS_{XY}$ and the price ratio is restored.
- If $MRS_{xy} < \frac{P_X}{P_Y} \Longrightarrow \frac{MU_X}{MU_Y} < \frac{P_X}{P_Y} \Longrightarrow \frac{MU_X}{P_X} < \frac{MU_Y}{P_Y}$ the consumer must increase the consumption of good Y (as he obtains greater per rupee utility from consumption of good Y) till the equality between the $MRS_{XY}$ and the price ratio is restored.

## 2.3.2 Marginal Rate of Substitution

An indifference curve is the locus of all possible commodity baskets that give the consumer the same level of utility. If U* is a fixed utility level, the equation of the indifference curve that corresponds to this utility level is given by

$$U(X, Y) = U^*$$

As the commodity baskets are changed by adding small amounts of X and subtracting small amounts of Y, the total change in utility must equal to zero along the indifference curve.

On total differentiating the above utility function we get,

$$\frac{\partial U}{\partial X} dX + \frac{\partial U}{\partial Y} dY = dU^*$$

Here, along an Indifference curve utility remains constant, that is, $dU^*$ the erefore,= 0.

Thus, we get $\qquad MU_X\, dX + MU_Y\, dY = 0 \qquad (8)$

where, $\qquad \frac{\partial U}{\partial X} = MU_X$ and $\frac{\partial U}{\partial Y} = MU_Y$

Rearranging,

$$-\frac{dY}{dX} = \frac{MU_X}{MU_Y} = MRS_{XY} \quad (9)$$

where $MRS_{XY}$ represents the individual's marginal rate of substitution of X for Y. Because the left-hand side of (9) represents the negative slope of the indifference curve, it follows that at the point of tangency, individual's $MRS_{XY}$ (which trades off goods while keeping utility constant) is equal to the ratio of marginal utilities of the two goods X and Y, which in turn is equal to the ratio of the prices of the two goods from Equation (7). When the individual indifference curves are convex, the tangency of the indifference curve to the budget line solves the consumer's optimisation problem.

### 2.3.3 Marginal Utility of Income

Whatever be the form of the utility function, the Lagrangian multiplier λ represents the extra utility generated when the budget constraint is relaxed— in this case by adding one rupee to the budget. To show how the principle works, we differentiate the utility function U(X, Y) totally with respect to M:

$$\frac{dU}{dM} = MU_X\left(\frac{dX}{dM}\right) + MU_Y\left(\frac{dY}{dM}\right) \qquad (10)$$

Substituting from (4) into (10), we get

$$\frac{dU}{dM} = \lambda P_X\left(\frac{dX}{dM}\right) + \lambda P_Y\left(\frac{dY}{dM}\right)$$

$$\frac{dU}{dM} = \lambda\left[P_X\left(\frac{dX}{dM}\right) + P_Y\left(\frac{dY}{dM}\right)\right] \qquad (11)$$

Because any increment in income must be divided between the two goods, by differentiating the budget equation with respect to income (M) it follows that

$$1 = P_X\frac{dX}{dM} + P_Y\frac{dY}{dM} \qquad (12)$$

Substituting Equation (12) in Equation (11), we get

$$\frac{dU}{dM} = \lambda(1) = \lambda$$

Thus, the Lagrangian multiplier is the extra utility that results from extra rupee of income.

### 2.3.4 Indirect Utility Function and Expenditure Function

When we plug the values of optimal quantities of both the goods, that is, X* and Y* (or Marshallian/ Walrasian/ ordinary demand functions) into the original utility function U(X, Y), we obtain the ***indirect utility function***—a function of prices and income only. That is,

$$V(P_X, P_Y, M) = U[X^*(P_X, P_Y, M), Y^*(P_X, P_Y, M)]$$

where, $V(P_X, P_Y, M)$ represents the indirect utility function, giving the maximum utility that can be achieved given the prices ($P_X$, $P_Y$) and income levels (M). It is indirect because while utility is a function of the commodity bundle consumed — X and Y, indirect utility function is a function of commodity prices and income—$P_X$, $P_Y$ and M.

Instead of maximising utility [U(X, Y)] subject to a given income (M) we can also minimise expenditure incurred on consumption of commodities (X and Y) subject to achieving a given level of utility $\overline{U}$. In this case, the consumer optimises by spending as little money as possible to enjoy a certain utility level. Formally, the optimisation exercise now becomes

$$\text{Minimise } P_x X + P_y Y$$

subject to the constraint of the given level of utility

$$U(X, Y) \geq \overline{U}$$

Solving the above optimisation problem using the method of Lagrangian like we did in the case of utility maximisation yields Compensated or Hicksian demand functions given by $X^*(P_X, P_Y, \overline{U})$ and $Y^*(P_X, P_Y, \overline{U})$, which are a function of prices $(P_X, P_Y)$ and given utility level $(\overline{U})$. On plugging the compensated demand functions into the objective function $(P_X X + P_Y Y)$, we obtain the ***expenditure function*** $E(P_X, P_Y, \overline{U})$—a function of prices $(P_X, P_Y)$ and given utility level $(\overline{U})$. It measures the minimal amount of money required to buy a bundle that yields a utility of $\overline{U}$.

**Example**

Consider a Cobb-Douglas utility function

$$U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

Assuming price of good $x_1$ and good $x_2$ to be $P_1$ and $P_2$, respectively and income be M. Determine the optimal choice of consumption of goods $x_1$ and $x_2$. Also find the expression for the indirect utility function.

**Solution**

Solving the problem using the equilibrium condition

$$MRS_{x_1 x_2} = \frac{P_1}{P_2}$$

$$MRS_{x_1, x_2} = -\frac{\partial U(x_1, x_2)/\partial x_1}{\partial U(x_1, x_2)/\partial x_2} = -\frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{x_1^{\alpha} \beta x_2^{\beta-1}}$$

$$= -\frac{\alpha x_1^{\alpha-1-\alpha}}{\beta x_2^{\beta-1-\beta}} \Rightarrow -\frac{\alpha x_2}{\beta x_1}$$

Now, $MRS_{x_1 x_2} = -\frac{\alpha x_2}{\beta x_1}$ and slope of the budget line is $-\frac{P_1}{P_2}$

Therefore, $-\frac{\alpha x_2}{\beta x_1} = \frac{-P_1}{P_2}$ or $-\alpha x_2 P_2 = -\beta x_1 P_1$

$$\frac{\alpha P_2 x_2}{\beta P_1} = x_1 \qquad (13)$$

Equation of the budget constraint is given by $P_1 x_1 + P_2 x_2 = M$

On inserting the value of $x_1$ from (13) in this constraint, we get

$$P_1\left(\frac{\alpha P_2 x_2}{\beta P_1}\right) + P_2x_2 = M$$

$$\frac{\alpha P_2 x_2}{\beta} + P_2x_2 = M \Longrightarrow \alpha P_2x_2 + \beta x_2P_2 = M\beta$$

$$\Longrightarrow (\alpha + \beta)x_2P_2 = M\beta$$

$$\Longrightarrow x_2 = \left(\frac{\beta}{\alpha+\beta}\right)\frac{M}{P_2} \quad (14)$$

Now, substituting value of $x_2$ from (14) in Equation (13) we get

$$x_1 = \frac{\alpha P_2 \frac{\beta}{\alpha+\beta}\frac{M}{P_2}}{\beta P_1} = \left(\frac{\alpha}{\alpha+\beta}\right)\frac{M}{P_1}$$

Therefore, $x_1 = \frac{\alpha}{\alpha+\beta} \cdot \frac{M}{P_1}$ and $x_2 = \frac{\beta}{\alpha+\beta} \cdot \frac{M}{P_2}$ are the ordinary or Walrasian or Marshallian demand functions.

For Indirect utility function, substitute the optimal values of $x_1$ and $x_2$ in the original utility function U ($x_1$, $x_2$) = $x_1^{\alpha}x_2^{\beta}$ . Thus, expression for Indirect utility function is given by,

$$V(P_1, P_2, M) = U[x_1(P_1, P_2, M), x_2(P_1, P_2, M)]$$

$$\Longrightarrow V(P , P_2, M) = \left[\left(\frac{\alpha}{\alpha+\beta}\right)\frac{M}{P_1}\right]^{\alpha} \left[\left(\frac{\beta}{\alpha+\beta}\right)\frac{M}{P_2}\right]^{\beta}$$

$$\Longrightarrow V(P_1, P_2, M) = \left(\frac{M}{\alpha+\beta}\right)^{\alpha+\beta} \left(\frac{\alpha}{P_1}\right)^{\alpha} \left(\frac{\beta}{P_2}\right)^{\beta}$$ is the required indirect utility function.

Suppose consumer purchase $x_1$* units of good $x_1$. Therefore, the total expenditure on $x_1$* will be $x_1$*$P_1$. The fraction of income spent on good $x_1$ shall be $\frac{x_1^* P_1}{M}$

We know $x_1^* = \frac{\alpha}{\alpha+\beta} \cdot \frac{M}{P_1}$.Therefore, $\left\{\left(\frac{\alpha}{\alpha+\beta}\right) \cdot \frac{M}{P_1} \cdot \frac{P_1}{M}\right\} = \left(\frac{\alpha}{\alpha+\beta}\right)$ will be the fraction of income spent on good $x_1$. Also, $\left\{\left(\frac{\beta}{\alpha+\beta}\right) \cdot \frac{M}{P_2} \cdot \frac{P_2}{M}\right\} = \left(\frac{\beta}{\alpha+\beta}\right)$ will be the fraction of income spent on good $x_2$. Thus, a consumer having Cobb-Douglas utility function always spends a fixed fraction of his income on each good.

**Check Your Progress 1**

1) Consumer A consuming goods $x_1$ and $x_2$ has utility function of form U($x_1$, $x_2$) = $4\sqrt{x_1}$ + $x_2$

 a) 'A' originally consumed 9 units of $x_1$ and 10 units of $x_2$. His consumption of $x_1$ is reduced to 4 units. After change, how much of $x_2$ should he be consuming to maintain same level of utility?

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b) If A is consuming the bundle (9, 20), what would be his MRS ($x_1$, $x_2$) and when is he consuming (9, 10)? Also, write MRS in general form.

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2) Utility function of an individual is given by U(X, Y) = $X^{3/4} Y^{1/4}$. Find out the optimal quantities of the two goods X and Y using Lagrangian method, if it is given that price of good X is Rs 6 per unit, price of good Y is Rs. 3 per unit and income of the individual is equal to Rs. 120.

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## 2.4 CONSUMER'S EQUILIBRIUM WITH INCOME TAX VS QUANTITY TAX

There are various economic policy tools (such as taxes, subsidies, rationing, etc.) that affect the budget constraint of the consumer. In this section, we shall study the impact of quantity tax and income tax on the consumer's equilibrium.

In case of a quantity tax, the consumer has to pay a certain amount to the government for each unit of the good purchased on which the tax is imposed. Suppose, before any quantity tax, the price of good X is $P_X$. Now the government imposes a quantity tax of Rs. t per unit on good X. This quantity tax changes the price of good from $P_X$ to $P_X + t$. Graphically, this would cause the budget line to become steeper. An income tax on the other hand is a tax on the income of the consumer. Graphically, this would cause the budget line to shift parallel leftwards as if the income has fallen.

**The analysis is based on the following assumptions:**

1) The consumer initially has a money income M, and the prices of the two goods X and Y are $P_X$ and $P_Y$ respectively.

2) The government has two economic policy tools to raise revenue:

   a) Quantity tax of Rs. t per unit on good X

   b) Income tax

3) The magnitude of revenue collected from taxes, is the same, either from quantity tax or income tax. The economic implication of this assumption is that the government is indifferent as to whether tax revenue is collected from quantity tax or income tax.

4) The consumer is rational guided by the objective of utility maximisation subject to the budget constraint.

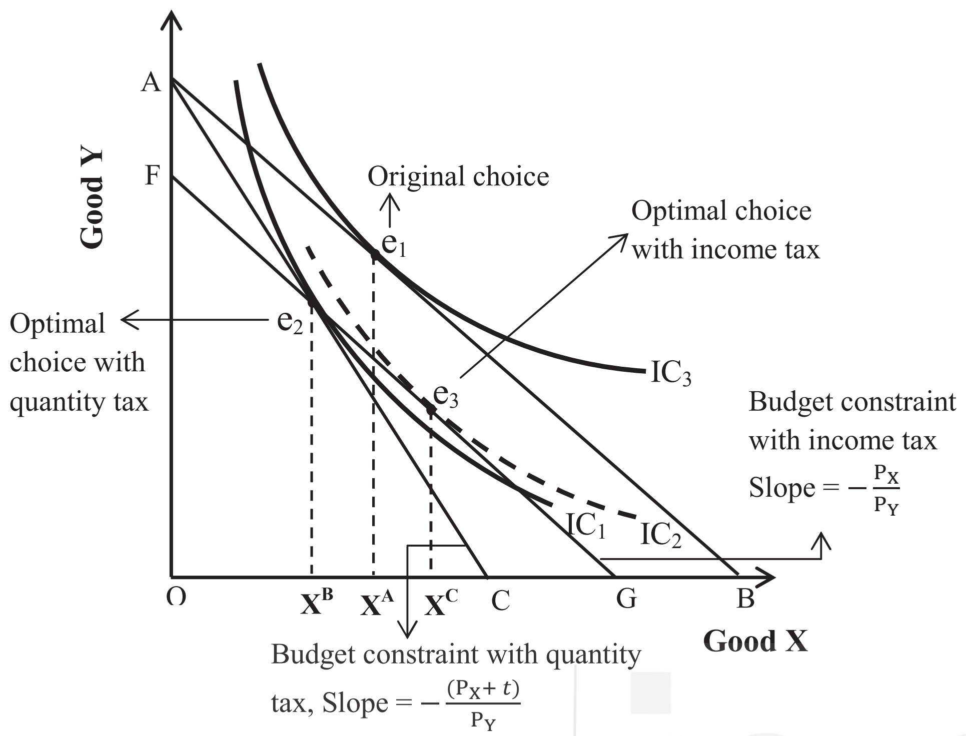


**Fig. 2.2: Consumer's Equilibrium with Income Tax vs Quantity Tax**

Now, consider Fig. 2.2 above. The consumer is initially in equilibrium at point $e_1$ where the budget line AB is tangent to $IC_3$. AB is defined by the equation

$$P_X X + P_Y Y = M \qquad (15)$$

and consumer consumes $OX^A$ units of good X and $e_1X^A$ units of good Y. Note that the coordinate of X intercept at the point B is $\left(\frac{M}{P_X}, 0\right)$, and that of Y intercept at the point A is $\left(0, \frac{M}{P_Y}\right)$. Now government imposes a quantity tax of Rs. t per unit on good X, so the post tax price of commodity X is $(P_X + t)$ with $P_Y$ being the price of commodity Y remaining unchanged. This causes the budget line to pivot leftwards from AB to AC. AC is defined by the equation

$$(P_X + t) X + P_Y Y = M \qquad (16)$$

The new equilibrium is established at $e_2$ where the budget line AC is tangent to $IC_1$. The consumer is consuming $OX^B$ units of good X and $e_2X^B$ units of good Y. The government is able to collect tax revenue R, equal to $tX^B$. At equilibrium $e_2$, we know

$$(P_X + t)X^B + P_Y (e_2X^B) = M \qquad (17)$$

As a policy option, if the government had imposed an income tax equal to $tX^B$, then the original budget constraint AB would have shifted parallel leftwards to FG, passing through point $e_2$. FG passes through $e_2$ because

$$R = tX^B \qquad (18)$$

Income after tax = $M - tX^B$

Therefore, equation of the new budget line becomes, $P_X X + P_Y Y = M - tX^B$, which has a slope of $-\frac{P_X}{P_Y}$ and will pass through ($X^B$, $e_2X^B$) as it will be a transformed form of Equation (17)⇒ $P_X X^B + P_Y(e_2X^B) = M - tX^B$ with an income tax, equilibrium is established at $e_3$ when the budget line FG is tangent to $IC_2$.

Now comparing a quantity tax with an income tax we find that the consumer is better off with an income tax. This is because with an income tax, consumer attained an equilibrium point $e_3$ on $IC_2$ and with a quantity tax he attained an equilibrium point $e_2$ on $IC_1$, and as you may notice, $IC_2$ provides a higher level of satisfaction than $IC_1$.

**Check Your Progress 2**

1) In a two-good world ($x_1$, $x_2$) with prices ($p_1$, $p_2$), assume that a consumer has a utility function $U = x_1^{1/2}.x_2^{1/2}$ with a budget constraint $p_1x_1 + p_2x_2 = M$. Initially, price of good $x_1$ is Rs. 2 and of good $x_2$ is Rs. 8. The income of the consumer is Rs. 400.

   a) Let the government impose a quantity tax of Rs. 2 per unit on good $x_1$. What happens to the optimal consumption bundles and find the amount of tax collected.

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   b) Suppose the government replaces the quantity tax by an equivalent income tax. Find the optimal consumption bundle. Is the consumer better or worse off compared to the quantity tax situation.

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2) Priya spends all her income of Rs. 5000 on food (F) and clothing (C). The price of food is Rs. 250 and that of cloth is Rs.100 and her monthly consumption of food is 10 units and that of clothing is 25 units. $MRS_{FC} = \frac{1C}{1F}$. Is she in equilibrium with this consumption, which commodity she will substitute for the other to reach equilibrium position?

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## 2.5 CONSUMER'S EQUILIBRIUM WITH CHANGE IN PRICE

In this section, we shall study the impact on consumer equilibrium when price of one of the goods changes. Considering a 2-good model, when price of one good, let say good X changes, this brings change in the price of good X (*i.e.*, $P_X$) relative to good Y, and also in the real income of the consumer. The overall change in quantity demanded of good X due to a change in its price, ceteris paribus, is called the price effect. This change can be further broken down into two components:

a) Substitution effect: Substitution implies a movement away from the relatively expensive good. Controlling for the change in real income, substitution effect captures the effect of change in the relative price ratio on the quantity demanded of the good whose price has changed.

b) Income effect, on the other hand measures the effect of change in the real income on the quantity demanded of the good whose price has changed.

This decomposition of the price effect into substitution and income effect can be done by way of two approaches:

i) Slutsky's approach

ii) Hicksian approach

### 2.5.1 Slutsky's Approach

We shall use Slutsky's method to break the price effect into income effect and substitution effect for good X wherein

Case 1 Good X is a normal good

Case 2 Good X is an inferior good

Case 3 Good X is a giffen good

**We make the following assumptions for all the three cases:**

1) The consumer is consuming two goods; good X and good Y.

2) The initial nominal income of the consumer is given by M.

3) The initial prices of the two goods are $P_X$ and $P_Y$.

4) All income is spent on the two goods and no part of the income is saved.

5) The initial budget line faced by the consumer is given by AB.

6) The consumer is initially in equilibrium at $e_1$, where the budget line AB is tangent to $IC_1$.

**Case 1:** Good $X_1$ is a normal good.

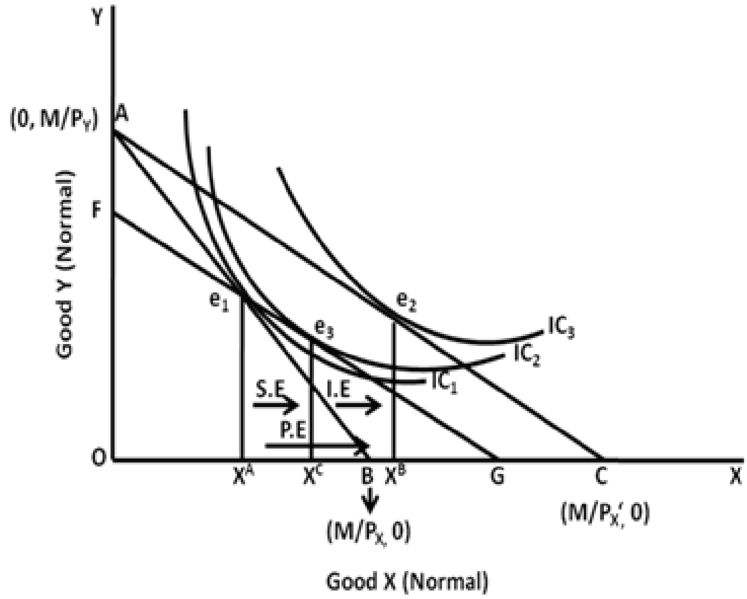


**Fig. 2.3: Decomposition of Price effect into Substitution effect and Income effect in case of Normal Good using Slutsky's approach**

Consider Fig. 2.3, where budget line AB is tangent to $IC_1$ and initial equilibrium is given by $e_1$, with consumer consuming $OX^A$ units of good X. Let the price of good X fall from $P_X$ to $P_X'$, ceteris paribus. This causes the budget line to pivot from AB to AC. The new equilibrium is established at $e_2$ where the budget line AC is tangent to $IC_3$. The consumer now purchases $OX^B$ units of good X. The total change in demand for good X is given by, $\Delta X = (OX^B - OX^A)$. This represents the magnitude of the price effect, which can be broken down into Substitution and the Income effect.

We know that when price of good X falls, the real purchasing power of the consumer rises $\left(\frac{M}{P_X} < \frac{M}{P_X'}\right)$. To eliminate this increase in real purchasing power, the money income of the consumer is reduced temporarily (say by taxation) by such an amount that the consumer is just able to afford his original preferred bundle (defined by $e_1$), at the new price ratio $\frac{P_X'}{P_Y}$. That is, allowing only for the change in the relative price ratio, while the real purchasing power is held constant. This is graphically achieved by introducing an income compensated budget line FG [defined by parameters ($P_X'$, $P_Y$, M')] parallel to AC and passing through $e_1$ (the original preferred bundle).

The consumer now attains equilibrium at point $e_3$ where FG is tangent to $IC_2$. The movement from $e_1$ to $e_3$ shows that the consumer increased his demand for good X from $OX^A$ to $OX^C$ as good X became relatively cheaper to good Y. Therefore, $OX^C - OX^A$ defines the substitution effect resulting from a change in the price of good X relative to good Y, with real income remaining constant.

To study the income effect, we now restore the money income of the consumer that was taken away. This is graphically achieved by shifting the budget line from FG to AC. The consumer moves from $e_3$ on $IC_2$ to $e_2$ on $IC_3$. Consumer increases his consumption of good X from $OX^C$ to $OX^B$. This increase in quantity demanded ($OX^B - OX^C$) measures the magnitude of the income effect.

We are now in a position to explain symbolically how the price effect is split into income and substitution effect.

1) $OX^A$ is calculated with parameters ($P_X$, $P_Y$, M)
2) $OX^B$ is calculated with parameters ($P_X'$, $P_Y$, M)
3) $OX^C$ is calculated with parameters ($P_X'$, $P_Y$, M')

Now, how to determine M'?

We know that point $e_1$ lies on two budget lines, AB and FG. There, the consumption bundle of good X and Y, given by ($OX^A$, $e_1X^A$) is the same for two budget lines AB and FG. Thus ($OX^A$, $e_1X^A$) is affordable at ($P_X$, $P_Y$, M) and ($P_X'$, $P_Y$, M'). That is, $e_1$ on AB shall satisfy the following equation

$$P_X X + P_Y Y = M \qquad (19)$$

and $e_1$ on FG shall satisfy the following equation

$$P_X' X + P_Y Y = M' \qquad (20)$$

Subtracting Equation (19) from Equation (20) i.e. [(20) – (19)], we get

$$M' - M = P_X' X - P_X X$$

$$M' - M = X (P_X' - P_X)$$

In simple words,

(New money income – original money income) = [(original quantity of good X demanded) multiplied by (new price – old price)]

Or $$\Delta M = X \Delta P_X$$

Remember:

ΔM and ΔP will always move in the same direction.

- If the price of a good falls, we shall need to reduce money income to keep the original bundle affordable at the new price ratio.
- If the price of a good rises, we shall need to increase money income of the consumer to keep the original bundle affordable, at the new price ratio.

Let us now symbolically represent the price effect (PE) as the sum of income effect (IE) and substitution effect (SE):

$$PE = SE + IE$$

$$\Delta X = \Delta X^S + \Delta X^N$$

where, $\Delta X$ = Total price effect

$\Delta X^S$ = the substitution effect

$\Delta X^N$ = the income effect

Now, $\Delta X = X (P_X', P_Y, M) - X (P_X, P_Y, M)$

$$\Delta X^S = X (P_X', P_Y, M') - X (P_X, P_Y, M)$$

$$\Delta X^N = X (P_X', P_Y, M) - X (P_X', P_Y, M')$$

We have $\Delta X = \Delta X^S + \Delta X^N$

$$X(P_X', P_Y, M) - X(P_X, P_Y, M) = [X(P_X', P_Y, M') - X(P_X, P_Y, M)] + [X(P_X', P_Y, M) - X(P_X', P_Y, M')]$$

The above equation is referred as the Slutsky's Identity. The above equation is an identity because the left hand side is equal to the right hand side as the first and the fourth terms on the right side cancel out.

**Case 2:** Good X is an inferior good

In case of an inferior good, increase in income of the consumer causes fall in the demand of that good. Good X is assumed to an inferior good. The initial equilibrium of the consumer is given by point $e_1$ on budget line AB, where AB is tangent to $IC_1$, with consumer consuming $OX^A$ units of good X. (Refer Fig. 2.4)

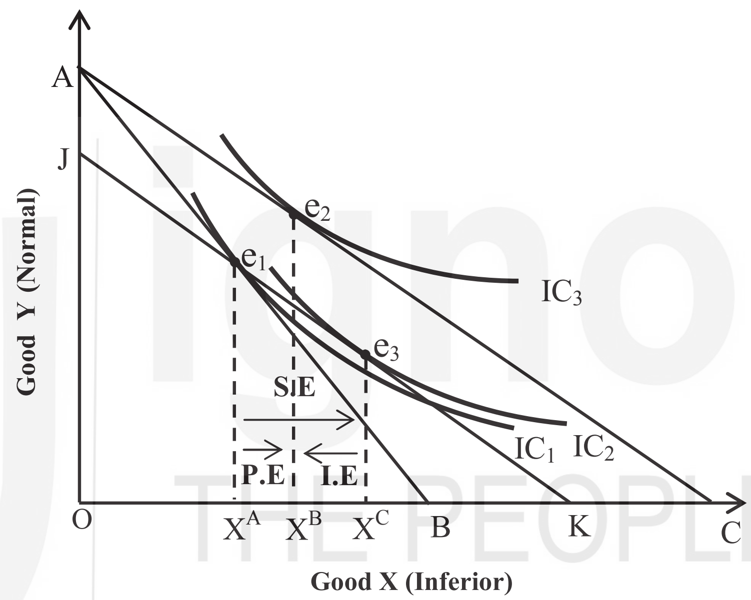


**Fig. 2.4: Decomposition of Price effect into substitution effect and Income effect in case of an Inferior Good using Slutsky's approach**

Let price of the inferior good X falls from $P_X$ to $P_X'$. This causes the budget line to pivot from AB to AC. Consumer reaches new equilibrium at $e_2$, purchasing $OX^B$ units of goods X. The total change in demand of good X, $\Delta X = (OX^B - OX^A)$ gives the magnitude of the price effect (notice that the law of demand holds true for an inferior good X).

Like we did in case of normal good, to cancel the income effect, we introduce an income compensated budget line JK parallel to AC (reflecting the new price ratio), by withdrawing some income of the consumer, so that the consumer can buy the original commodity bundle at the new price (hence JK is passing through $e_1$). At new equilibrium $e_3$, given by the tangency of JK and $IC_2$, consumer purchases $OX^C$ units of good X. Movement from $e_1$ to $e_3$ shows that the consumer increased his demand for the inferior good X from $OX^A$ to $OX^C$ by substituting the dearer commodity Y for the cheaper commodity X giving the magnitude of the substitution effect as $OX^C - OX^A$. On restoring the money income of the consumer, equilibrium moves from $e_3$ on $IC_2$ to $e_2$ on $IC_3$. The consumer decreases his consumption of good X from $OX^C$ to $OX^B$ under the income effect as good X is an inferior good.

You will notice that while the substitution effect causes an increase in quantity demanded (from $OX^A$ to $OX^C$), the income effect causes a decrease in the quantity demanded (from $OX^C$ to $OX^B$). You can further see that the strength of the substitution effect is relatively stronger than the strength of the income effect. The final result (of price effect) from the sum of the substitution effect and income effect is that there is still rise in quantity demanded ($OX^A$ to $OX^B$) for the inferior good X when its prices have decreased from $P_X$ to $P_X$'. Thus, the law of demand operates even in case of inferior goods.

**Case 3:** Good X is a Giffen good

It was observed by Sir Robert Giffen, a British economist of the $19^{th}$ century, that as the price of bread rose in England, many low paid workers began to purchase more bread. This observation was contrary to the law of demand. A good which behave contrary to the law of demand is known as a "Giffen-type" good and the phenomenon as "Giffen's Paradox"*. The cardinal utility analysis could not explain this "Giffen's Paradox", as it did not treat the price effect as a combination of substitution and income effect. It completely ignored the income effect of a price change, by assuming constant marginal utility of money.* Indifference curve analysis has been successful in overcoming this limitation of cardinal utility analysis. It has been able to resolve the "Giffen's paradox". Even in case of a "Giffen" good, the substitution effect causes an increase in quantity demanded for a fall in price of the Giffen good, but simultaneously the income effect acts to reduce the quantity demanded as all Giffen goods are inferior goods. However in this case, the dominance of income effect over the substitution effect makes the price effect positive (fall in price of good X is accompanied by rise in quantity demanded) leading to the violation of law of demand. This is explained with the help of Fig. 2.5 below. Let us assume that good X is a Giffen good. The consumer is initially in equilibrium at point $e_1$, purchasing $OX^A$ units of good X.

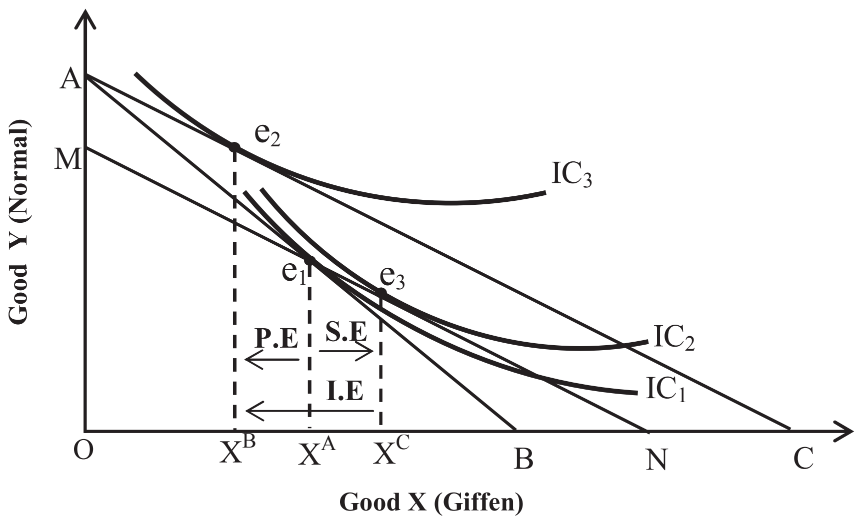


**Fig. 2.5: Decomposition of Price effect into substitution effect and Income effect in case of a Giffen Good using Slutsky's approach**

With a fall in price of good X (which is a Giffen good), the budget line pivots from AB to AC, with consumer reaching equilibrium at $e_2$. The consumer now demands a lower quantity of good X, although he/she is on a higher indifference curve. The quantity decreases by ($OX^B$ – $OX^A$), giving the price effect.

Now, using the same methodology of separating substitution effect from income effect, we draw income compensated budget line MN parallel to AC, passing through $e_1$. The new equilibrium is established at point $e_3$ on $IC_2$, with consumption of good X increased to $OX^C$. The increase in quantity demanded ($OX^C$ – $OX^A$) defines the substitution effect. To study the income effect, the money income that was taken away from the consumer is restored. The consumer moves from point $e_3$ on $IC_2$ to $e_2$ on $IC_3$. The consumer reduces his consumption from $OX^C$ to $OX^B$ because Giffen good X is an inferior good. The decrease in quantity demanded ($OX^C$ – $OX^B$) measures the income effect.

While the substitution effect caused an increase in quantity demanded for a fall in price, the income effect caused a substantial decrease in quantity demanded. In case of a Giffen good, the strength of the income effect is so strong that it outweighs the substitution effect, causing a decrease in quantity demanded from $OX^A$ to $OX^B$, with a fall in price of good X. Thus, the "Giffen's paradox" is a strong exception to the law of demand.

### 2.5.2 Hicksian Approach

In Slutsky's approach, substitution effect is given by the change in demand of the good whose price changes, holding constant consumer's real income. This we measured by constructing an income compensated budget line (by introducing a compensated change in income of the consumer opposite to the change in price) parallel to the pivoted budget line after the price change, reflecting new price ratio but compensated income and passing through the original equilibrium bundle.

Hicksian approach, on the other hand, involves measurement of the substitution effect as the change in the demand of the good whose price has changed, holding utility constant. Now, this will involve constructing the income compensated new budget line reflecting new price ratio, parallel to the pivoted budget line and tangent to the original indifference curve. This is the point of difference between the Slutsky's and the Hicksian approach, while the former requires keeping constant the initial purchasing power (so that the consumer can buy the initial commodity bundle at the new price), the latter involves keeping constant the original utility level (so that the consumer can attain the initial utility level at the new price), but both the approaches involve altering the money income of the consumer after the price change to separate substitution effect from the income effect. Let us graphically present the Hicksian approach (refer Fig. 2.6).

We assume the same case-I that we did under Slutsky's approach in Fig. 2.3. Consumer's initial equilibrium is given by $e_1$, where budget line AB is tangent to $IC_1$, with consumer consuming $OX^A$ units of good X. Let the price of good X fall from $P_X$ to $P_X'$, ceteris paribus. This causes the budget line to pivot from

AB to AC. At new equilibrium $e_2$ consumer consumes $OX^B$ units of good X. The total change in demand for good X is given by, $\Delta X = (OX^B - OX^A)$. This represents the magnitude of the price effect.

Now as per Hicksian approach, substitution effect is separated from the income effect by drawing income compensated budget line JK parallel to AC, but tangent to the original indifference curve, $IC_1$ (Refer Fig. 2.6). That is, after the fall in the price of good X, money income is reduced to the extent that consumer can afford the previous utility level. Equilibrium $e_2$, given by the tangency of JK and $IC_1$, marks the change in demand of good X resulting from change in the price ratio, keeping constant the utility level. Thus, substitution effect is given by $OX^C - OX^A$. The remaining change in the quantity demanded of good X from $OX^C$ to $OX^B$ account for the income effect.

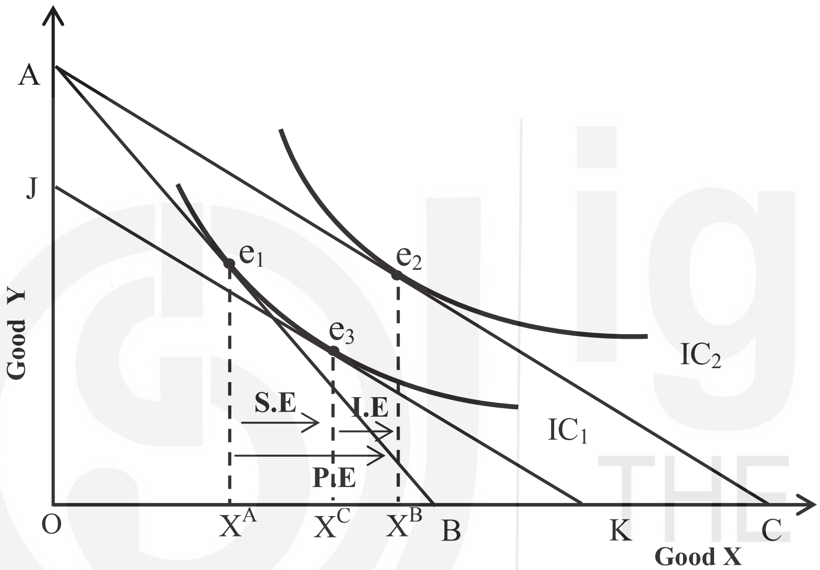


**Fig. 2.6: Decomposition of Price effect into substitution effect and Income effect in case of a Normal good using Hicksian approach**

The other cases, of that of an inferior good and a Giffen good, can be similarly analysed.

### 2.5.3 Estimation of Substitution and Income Effect through Slutsky's and Hicksian Approach

Consider two goods, X and Y, priced at $P_X$ and $P_Y$. Let M be the income of the consumer. Initial demand for both the goods will be a function of $(P_x, P_Y, M)$, given by

$$X^O (P_X, P_Y, M) \text{ for good X and } Y^O (P_X, P_Y, M) \text{ for good Y}$$

Now let price of good X fall from $P_X$ to $P_X'$. Final demand for the both the goods will be a function of $(P_X', P_Y, M)$, given by, $X^F (P_X', P_Y, M)$ for good X and $Y^F (P_X', P_Y, M)$ for good Y. Now, price effect equals, $X^F (P_X', P_Y, M) - X^O (P_X, P_Y, M)$. This can be further separated into substitution and income effect by Slutsky's and Hicksian approach.

i) **Slutsky's Approach**

This approach involves finding out the intermediate demand for good X at the new price ratio $\frac{P_X'}{P_Y}$, keeping constant the original purchasing power or the real income of the consumer. Let $M_s$ be the altered money income spending which consumer can consume the original bundle of both the goods i.e., $(X^O, Y^O)$ after $P_X$ changes to $P_X'$. That is, we have

$$M_s = P_X' X^O + P_Y Y^O$$

This represents equation of the income compensated budget line, which we draw parallel to the pivoted budget line, after passing through the original consumption bundle. Now, optimal consumption of good X on this new budget line will be a function of $(P_X', P_Y, M_s)$, let us denote it by, $X^S (P_X', P_Y, M_s)$, with $M_s$ itself given by $(P_X', P_Y, X^O, Y^O)$, we can also write $X^S (P_X', P_Y, X^O, Y^O)$.

Now, Substitution effect $= X^S - X^O$

and Income effect $= X^F - X^S$

ii) **Hicksian Approach**

The sole difference between the Slutsky's and Hicksian approach is the estimation of the intermediate demand. Hicksian approach involves finding out the intermediate demand for good X at the new price ratio $\frac{P_X'}{P_Y}$, keeping constant the original utility level of the consumer. Let $M_h$ be the altered money income spending which consumer can attain the original utility level (let say $U^O$) after $P_X$ changes to $P_X'$. That is, we have

$$U^O = U\,[X(P_X', P_Y, M_h), Y(P_X', P_Y, M_h)]$$

where utility function is given by U(X, Y), with X and Y themselves being a function of $(P_X', P_Y, M_h)$. The optimal demand for good X associated with income $M_h$ is given by $X^h (P_X', P_Y, M_h)$, where $M_h$ itself is a function of $(P_X', P_Y, U^O)$, thus we can write $X^h (P_X', P_Y, U^O)$.

Now, Substitution effect $= X^h - X^O$

and Income effect $= X^F - X^h$

**Example**

Suppose consumer 'A' has utility function of the form $U(x_1, x_2) = x_1x_2$. Let price of good $x_1$ be $P_1$ = Rs. 2 and good $x_2$ be $P_2$ = Re 1. Now let price of $x_1$ falls from Rs. 2 to Re 1. A's income is Rs. 40/day. Answer the following:

a) Before the price change what was A's consumption bundle?

b) After the price change, if A's income had changed, so that he could afford old bundle exactly, what would A's income be? What is the consumption bundle with new income and price?

c) Break up the price effect into substitution effect and income effect.

**Solution**

a) Using the equilibrium condition $\frac{P_1}{P_2} = \text{MRS}_{12}$, we can determine A's consumption function before the price change.

We have $\text{MRS}_{12} = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{x_2}{x_1}$ and $\frac{P_1}{P_2} = \frac{2}{1}$, for equilibrium, $\frac{x_2}{x_1} = \frac{2}{1} \Longrightarrow x_2 = 2x_1$ (21)

Using Equation (21) and the budget constraint, given by $40 = 2x_1 + x_2$, we get $x_1^0 = 10$, $x_2^0 = 20$.

b) Going by the Slutsky's approach, income required to exactly afford original bundle would be given by, $M_s = p_1'x_1^0 + p_2x_2^0$, where $p_1' = 1$, $x_1^0 = 10$, $p_2 = 1$, $x_2^0 = 20$. Thus, we get $M_s = 30$.

Using equilibrium condition, $\frac{x_2}{x_1} = 1 \Longrightarrow x_2 = x_1$ and the new budget constraint given by $30 = x_1 + x_2$, we get the consumption levels with new income ($M_s = 30$) and price ($p_1' = 1$), given by $x_1^S = 15$ and $x_2^S = 15$.

c) Now, in order to break up price effect into substitution effect and income effect, we need to derive demand for good $x_1$ after its price changes, keeping constant all the other parameters. Using equilibrium condition $\frac{x_2}{x_1} = \frac{P_1'}{P_2} \Longrightarrow \frac{x_2}{x_1} = 1 \Longrightarrow x_2 = x_1$ and pivoted budget line equation, $40 = x_1 + x_2$, we get $x_1' = 20$ and $x_2' = 20$.

Now, Substitution effect $= x_1^s - x_1^0 = 15 - 10 = 5$

and Income effect $= x_1' - x_1^s = 20 - 15 = 5$

## 2.6 CONSUMER'S EQUILIBRIUM UNDER SPECIAL CIRCUMSTANCES

### 2.6.1 Perfect Complements

We now consider a case where good X and good Y are always consumed together in a fixed proportion. Thus, good X and good Y are perfect complements.

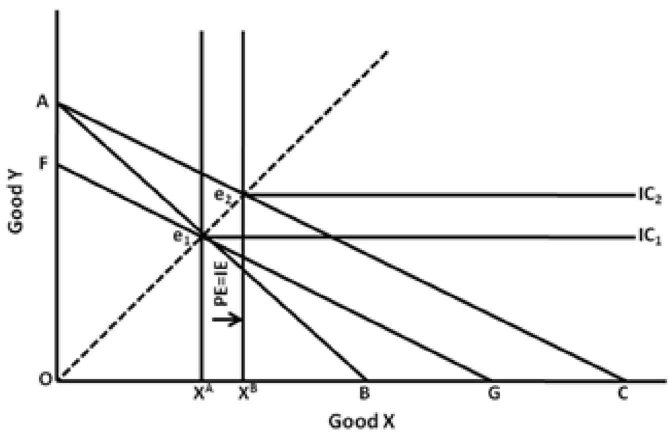


**Fig. 2.7**

In Fig. 2.7, the consumer is initially in equilibrium at point $e_1$ where the budget line AB is tangent to the L-shaped $IC_1$. The consumer is initially consuming $OX^A$ units of good X and $e_1X^A$ units of good Y. The budget line is defined by parameters ($P_X$, $P_Y$, M).

Let the price of good X fall from $P_X$ to $P_X'$, ceteris paribus. This causes the budget line to pivot rightwards from AB to AC. The new equilibrium is established at point $e_2$ where the budget line AC becomes tangential to $IC_2$. The consumer now consumes $OX^B$ units of good X and $e_2X^B$ units of good Y. The change in quantity demanded of good X due to a change in its price is the magnitude of the price effect which equals $OX^B – OX^A$.

We now break up the price effect into substitution and income effect. We first cancel the income effect by reducing the nominal income of the consumer from M to M' such that the real purchasing power is held constant and the original preferred bundle $e_1$ is just affordable. This is graphically achieved by shifting the budget line AC parallel downwards to FG till it passes through $e_1$. The new equilibrium is once again established at $e_1$ when FG is tangent to $IC_1$. This means that the magnitude of the substitution effect is zero. Intuitively, we can say that the substitution effect is zero as good X and good Y are perfect complements and there is no possibility of substitution of one good by the other complementary good.

We now restore the money income of the consumer that had earlier been taken away to measure the magnitude of the income effect. This is graphically achieved by shifting the budget line from FG to AC. The equilibrium is established at $e_2$ on $IC_2$. The change in demand due to the income effect is $OX^B – OX^A$. Therefore, the entire price effect is equal to the income effect (because substitution effect is zero). Thus, in case of perfect compliments, price effect equals income effect and substitution effect equals zero.

## 2.6.2 Perfect Substitutes

Let good X and good Y be perfect substitutes with $MRS_{XY}$ = 1 (in absolute terms). This implies that one unit of X can be equally well replaced by one unit of Y or vice-versa. This further implies that the indifference curves shall be linear (making a 45$^o$ angle with the axes).

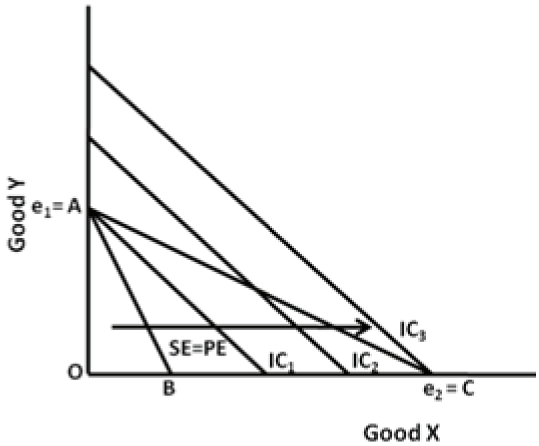


Fig. 2.8

In Fig. 2.8, let $P_X$ be greater than $P_Y$ initially. This implies that the budget line (drawn as AB) shall be steeper than the indifference curves ($IC_1$, $IC_2$ etc) because if $P_X > P_Y$, then slope of AB, given by $P_X/P_Y$ > slope of IC's (which equals 1). The consumer shall initially be in equilibrium at point A = $e_1$ (on the y-axis) only consuming good Y. (Intuitively, the consumer shall not purchase any units of X as X and Y are perfect substitutes and the price of good X is greater than price of good Y).

Now let price of good X fall from $P_X$ to $P_X'$ such that $P_X'$ falls below $P_Y$. This causes the budget line to pivot from AB to AC. As you may notice, new budget line AC is now flatter than the indifference curves, because if $P_X' < P_Y$, then $P_X'/P_Y$ (slope of AC) < 1 (slope of IC's). New equilibrium is established at point C = $e_2$. The consumer now purchases only good X and zero units of good Y. This is because $P_X' < P_Y$ and X and Y are perfect substitutes. The consumer was initially purchasing zero units of good X and after the price change the consumer purchases OC units of good X. Therefore, the total change in demand for good X due to the change in price is OC. OC is therefore the magnitude of the price effect (PE) = $e_2 - e_1$ = OC – 0 = OC.

In order to decompose the price effect into substitution effect and income effect we first eliminate the income effect. This would be attained graphically by shifting the budget line parallel downwards till the original bundle (in this case point A) is just affordable. Since point A lies on AC we cannot shift the budget line parallel downwards. This implies that income effect shall be zero. As the budget line after cancelling the income effect stays at AC, the magnitude of the substitution effect is OC, which is also equal to the price effect. In case of perfect substitutes, the total change in demand is due to the substitution effect and income effect is zero.

## 2.6.3 Quasi Linear Preferences

Let the consumer face quasi linear preferences and his utility function is defined by $U(X, Y) = V(X) + Y$. The indifference curve map faced by the consumer is drawn in the Fig. 2.9 below.

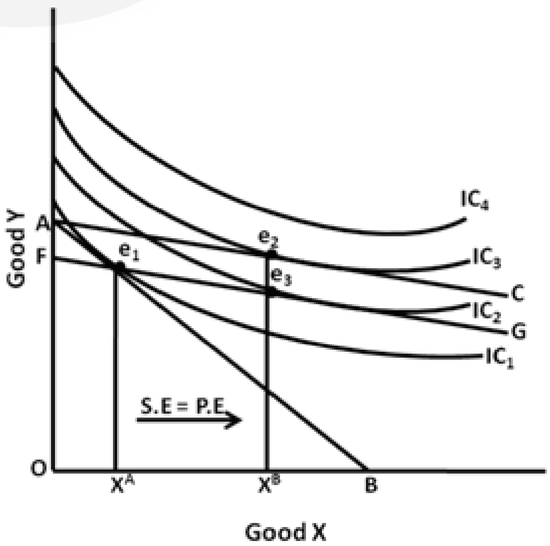


Fig. 2.9

The budget line AB is defined by parameters ($P_X$, $P_Y$, M). The consumer is initially in equilibrium at $e_1$ when the budget line AB is tangent to $IC_1$. The consumer is initially consuming $OX^A$ units of good X and $e_1X^A$ unit of good Y. Let price of good X fall from $P_X$ to $P_X'$, ceteris paribus. This causes the budget line to pivot from AB to AC. The new equilibrium is established at $e_2$ where AC is tangent to $IC_3$. The consumer, after the price change, is consuming $OX^B$ units of good X and $e_2X^B$ units of good Y. The total change in demand for good X is $OX^B - OX^A$ (which is the magnitude of the price effect) resulting from the price change.

We now break up the price effect into income and substitution effects using Slutsky's method. We first eliminate the income effect by reducing the money income of the consumer to an extent that the consumer is just able to afford the original preferred bundle at the new price ratio. This is graphically achieved by shifting the budget line parallel downwards from AC to FG, with FG passing through point $e_1$ (ensuring that the original preferred bundle is still affordable). The new equilibrium is established vertically below $e_2$ at $e_3$. This is because each indifference curve is a vertical translate of the original indifference curve, and with the budget line also shifting parallel downwards, there is no option but for the new equilibrium to be established at $e_3$ on $IC_2$.

Having eliminated the income effect, the consumer now consumes $OX^B$ units of good X and $e_3X^B$ units of good Y. Therefore, the magnitude of the substitution effect is $OX^B - OX^A$. We now restore the money income to measure the magnitude of income effect. This is graphically achieved by shifting the budget line from FG to AC. The consumer moves from $e_3$ on $IC_2$ to $e_2$ on $IC_3$. There is no change in the consumption of good X. Therefore, the magnitude of income effect of the price change on good X is zero. The entire magnitude of the price effect is the substitution effect. Thus, in case of quasilinear preferences, price effect equals substitution effect and income effect is zero.

**Check Your Progress 3**

1) Consider a consumer with the utility function given by, U (X, Y) = XY, where X and Y represent the two goods of consumption, priced at $P_x$ and $P_Y$, respectively. Assuming income of this consumer to be Rs. 120, $P_x$ = Rs. 3 and $P_Y$ = Re. 1.

   a) Find the equilibrium quantities of consumption of both the goods.

   b) Suppose price of good X fall to Rs. 2.5, what will be the impact on consumption quantities of both the goods?

   c) Estimate the price effect of the price fall on consumption of good X.

   d) Decompose the price effect into substitution and income effect for good X.

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2) A spends all his income on $x_1$ and $x_2$. According to him, $x_1$ and $x_2$ are perfect substitutes. Given, $P_1$ = Rs. 4, $P_2$ = Rs. 5, answer the following:

a) Suppose price of $x_1$ falls to Rs. 3, will its quantity demanded increase? Why?

b) Now suppose $P_2$ falls to Rs. 3 and $P_1$ does not change. What happens to its quantity demanded and why? Represent the situation graphically.

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3) Two goods are perfect complements. If price of one good changes, what part of change in demand is due to income effect (IE) and what part is due to substitution effect (SE)?

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## 2.7 LET US SUM UP

The theory of consumer choice is designed to explain how and why consumers purchase the combination of goods and services they do. The theory emphasises on two factors: the consumer's budget line, which shows the market baskets that can be bought; and the consumer's preferences, which indicate the subjective ranking of different market baskets. The effect of a price change on the quantity demanded of a good can be broken into two parts: a substitution effect, in which the level of real income remains constant while price changes, and an income effect, in which the price remains constant while the level of real income changes. Because the income effect can be positive or negative, a price change can have a small or a large effect on quantity demanded. In the unusual case of a so-called Giffen good, the quantity demanded may move in the same direction as the price change thereby generating an upward-sloping individual demand curve.

We began by explaining the theory of consumer's behaviour, i.e., the explanation of how consumers allocate incomes to purchase different goods and services. Further, we explained using indifference curve analysis that the consumer shall be better off under an income tax vis-à-vis a commodity tax, given the assumption that the revenue collected from both types of taxes is the same for the government. The discussion ended on splitting up price effect into income effect and substitution effect using Slutsky's and Hicksian methods.

## 2.8 REFERENCES


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## 2.9 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

**Check Your Progress 1**

1) a) $x_2 = 14$ units

   b) $MRS x_1x_2 = \frac{MU_1}{MU_2}$; MRS (9, 20) = $\frac{-2}{\sqrt{9}} = \frac{-2}{3}$; MRS (9, 10) = $\frac{-2}{\sqrt{9}} = \frac{-2}{3}$

   Therefore, MRS is indifferent of the value of $x_2$.

2) x = 15, y = 10

   Lagrangian expression for the given problem will be

   $L = x^{3/4}y^{1/4} + \lambda\,(120 - 6x - 3y)$

   Equation condition will be $\frac{\frac{3}{4}x^{-1/4}y^{1/4}}{\frac{1}{4}x^{3/4}y^{-3/4}} = \frac{6\lambda}{3\lambda}$

**Check Your Progress 2**

1) a) Given: $U = x_1^{1/2}\ x_2^{1/2}$

   Post tax optimal bundle: (50, 25)

   Amount of tax collected = Rs 100

   **Hint:** At the optimum consumption bundle,

   $$\text{MRS}\ x_1x_2 = \frac{P_1}{P_2}$$

   Taking the log transformation of the utility function, we get, V = ln (U)

   $$V = \frac{1}{2}\ln x_1 + \frac{1}{2}\ln x_2$$

   $$MV_1 = \frac{1}{2} \times \frac{1}{x_1} \quad \text{and} \quad MV_2 = \frac{1}{2} \times \frac{1}{x_2}$$

   Now, $\frac{MV_1}{MV_2} = \frac{P_1}{P_2}$

   $$\frac{\frac{1}{2}\times\frac{1}{x_1}}{\frac{1}{2}\times\frac{1}{x_2}} = \frac{P_1}{P_2} \Rightarrow \frac{x_2}{x_1} = \frac{P_1}{P_2} \Rightarrow x_2 = \frac{x_1P_1}{P_2}$$

   On solving we will get pre tax bundle $(x_1^*, x_2^*) = (100, 25)$

Now, the government imposes a quantity tax of Rs. 2 per unit on good $x_1$

Now $P_1 = 4$, $\therefore x_1^* = \frac{400}{2 \times 4} = 50$ $\therefore$ Post tax bundle (50, 25)

The amount of tax collected = 2 × 50 = Rs 100

b) New Bundle: (75, 18.75) consumer is better off

**Hint:** Income tax = 100

M = 400 – 100 = 300

$x_1^* = \frac{300}{2 \times 2} = 75$, $x_2^* = \frac{300}{2 \times 8} = 18.75$

| Original bundle | Quantity tax bundle | Income tax bundle |
|---|---|---|
| (100, 25) | (50, 25) | (75, 18.75) |

Original utility $U_0 = (100)^{1/2} (25)^{1/2} = 10 \times 5 = 50$

Utility after quantity tax $U_1 = (50)^{1/2} (25)^{1/2} = 7.07 \times 5 = 35.35$

Utility after income tax $U_2 = (75)^{1/2} (18.75)^{1/2} = 8.66 \times 4.33 = 37.49$

$\therefore U_2 > U_1$ , meaning that the consumer is better off under income tax compared to the quantity tax situation.

2) Given, $MRS_{FC} = 1$

Also, $\frac{P_F}{P_C} = \frac{250}{100} = 2.5$

$\Longrightarrow MRS_{FC} < \frac{P_F}{P_C}$ But for equilibrium, $\frac{P_F}{P_C} = MRS_{FC}$

Therefore, presently she is not in equilibrium position. Now, since in the market with the given prices of the two goods, she can exchange 1 unit of F for 2.5 units of C while she is willing to forego 1 unit of F for 1 unit of C to keep her satisfaction constant, she will be increasing her satisfaction *by consuming less of food and substituting C for it.* This is because with loss of 1 unit of food she will be having extra 2.5 units of C, while only one unit of C is sufficient to compensate her for the loss of 1 unit of F.

**Check Your Progress 3**

1) a) $X^0 = 20$ and $Y^0 = 20$

**Hint:** We use the equilibrium condition, $\frac{P_X}{P_Y} = MRS_{XY}$, where $P_X = 3$, $P_Y = 1$, MRS = $\frac{Y}{X}$ and the budget line equation, 120 = 3X + Y to arrive at $X^0 = 20$ and $Y^0 = 60$ (original bundle).

b) X' = 24 and Y' = 60

**Hint:** We use the equilibrium condition, $\frac{Y}{X} = \frac{P_X'}{P_Y}$, where $P_X' = 2.5$, $P_Y = 1$ and the budget line equation, 120 = 2.5X + Y to arrive at X' = 24 and Y' = 60 (final bundle).

c) Price effect = $X' - X^0 = 24 - 20 = 4$

d) Substitution effect = 2 and Income effect = 2

2) a) Yes. It is given that $x_1$ and $x_2$ are perfect substitutes for A. $x_1$ is cheaper. So A would consume only $x_1$. When price of $x_1$ falls further to Rs. 3 and with same money income he would be able to buy more of $x_1$. Hence, A starts to consume more of $x_1$. It is due to income effect.

b) Refer Sub-section 2.6.2 and draw

**Hint:** As $P_2$ falls to Rs. 3 and $x_1$ and $x_2$ being perfect substitute, A will spend his entire income to buy $x_2$ as $P_2 < P_1$.

New budget line: $4x_1 + 3x_2 = 120$

When $x_2 = 0$, $x_1 = 30$ and $x_1 = 0$, $x_2 = 40$. Hence, New equilibrium bundle is (0, 40)

3) In this case entire change is due to income effect. So, PE = IE and SE=0.

# UNIT 3 CONSUMER'S SURPLUS

**Structure**



## 3.0 OBJECTIVES

After going through this unit, you will be able to:

- get an insight into the concept of Consumer's Surplus;
- explain the concept of Consumer's Surplus for a discrete and a non-discrete goods;
- estimate Change in Consumer's Surplus resulting from price changes in case of observable demand curves;
- get an introduction to the concept of Quasi-linear utility and analyse it as the measure of a 'Change in consumer's surplus';
- discuss the three measures of change in utility *viz*. change in consumer's surplus, equivalent variation and compensating variation; and
- identify the relationship between the three measures of change in utility.

## 3.1 INTRODUCTION

Several factors affect market participants as they are present in the market for utility gains. Any change in the market conditions, for instance, change in the prices of the commodities available for sale or purchase, or change in the resources available with the participant, has an effect on the participant's behaviour and utility, be it the consumer of the commodity or the producer of it. In the previous units, we have discussed in detail about the consumer theory, wherein, the behaviour of the consumer as the participant in the market for utility gains was explained. You were introduced to the concepts of demand theory in the earlier course entitled, "Introductory Microeconomics" in semester one. Followed by this, the learner got insight into the consumer's behaviour and attainment of equilibrium through Cardinal utility and Ordinal utility approaches.

A consumer participates in the market with the main objective of getting maximum satisfaction from spending his given income on various goods and services. The satisfaction he attains results not only from the consumption of the goods or services but also from the gains that arise when he ends up paying less than the amount he was willing to pay for consumption of the good or service. In simple terms, this is what is referred to as the consumer's surplus.

In this unit, with the help of diagrams, we will discuss the concept of consumer's surplus, in case of both discrete and non-discrete goods. This discussion will be based on Marshallian demand curves, where surplus to a consumer will be measured by his demand curve for good or service and the current market price. Followed by this, we shall explain the effects of a change in price on the consumer's surplus. We shall further discuss the concept of Quasi-linear utility and the underlying connection between the consumer's surplus and the change in utility in case of quasi-linear preferences. Refuting the assumption of constant marginal utility of money as was assumed under Marshallian approach, you will also be introduced to two alternative measures of change in utility *viz*. Compensating variation and Equivalent variation introduced by John R. Hicks. In the end, we shall explain the relationship between the three measures of utility change, of which consumer's surplus is just one part. You will observe that only in case of quasi-linear preferences, the three measures (that is, change in consumer's surplus, equivalent variation and compensating variation) are equal.

## 3.2 THE CONCEPT OF CONSUMER'S SURPLUS

Alfred Marhall in his famous book "Principles of Economics" (first published in 1890) introduced, among other things, a theory of consumer and producer behaviour, derived demand and supply curves and used them in partial equilibrium analysis. The concept of Consumer's surplus was an integral part of this analysis. In order to understand the concept of Consumer's surplus, also known as Buyer's surplus, let us consider a hypothetical market situation. Suppose there is a commodity 'X' in the market, and you intend to participate in the market as a buyer. You are willing to pay Rs. 50 for one unit of commodity X based on its worth to you. However, as you enter the market and enquire about its price from the seller, you get to know that the price of the commodity is Rs. 25. Here, the difference between what you are willing to pay for commodity X and the actual market price of that commodity (Rs. 50 – Rs. 25 = Rs. 25 in our example) is called consumer's or buyer's surplus. It measures the utility gain to the group or individuals who purchase a particular good (commodity X here) at a particular price (Rs. 25 here).

Now, we come to a formal definition of Consumer's Surplus— *Consumer's surplus is defined as the difference between what consumers are willing to pay for a unit of the commodity and the amount the consumers actually pay for that commodity.* Willingness to pay can be read of as an individual or a market demand curve for a product. The market demand curve shows the quantity of the good that would be demanded by all consumers at each and every price that might prevail. Read the other way: the demand curve tells us the maximum price that consumers would be willing to pay for any quantity supplied to the market.

An illustration of the consumer's surplus can be derived by considering the following exercise. Consider Table 3.1, with four potential buyers (A, B, C and D) and their willingness to pay, representing the maximum amount a buyer will pay for a good.

**Table 3.1: Buyers with their willingness to pay**

| Buyers | Willingness to Pay (Rs) |
|---|---|
| A | 2000 |
| B | 1700 |
| C | 1500 |
| D | 1200 |

If the market price of the good is Rs. 1200 per unit, then A earns consumer's surplus of Rs. 800, since he was willing to pay Rs. 2000, but only had to pay Rs. 1200. Similarly, B earns Rs. 500 of consumer's surplus, and C earns Rs. 300 of consumer's surplus. Buyer D is willing to pay Rs. 1200 for a unit, but since the market price is Rs. 1200, D gets no consumer's surplus; hence he is the so-called "marginal" buyer.

## 3.3 CONSUMER'S SURPLUS AND THE DEMAND CURVE

### 3.3.1 Consumer's Surplus for a Discrete Good

Let us begin our estimation of the consumer's surplus from the case of a discrete good — that is, one that can be bought and sold only in integer units. Here, you should be clear about the concept of a *reservation price* as well. The maximum price that the consumer is willing to pay for a unit of a good than go without, is the reservation price for the unit purchased. In simple words, it is the monetary value of the one unit of a good consumed. It measures the marginal utility to the consumer from the purchase and consumption of a unit of the good. Symbolically, reservation price $r_n$ (per unit) is the price at which consumer is indifferent between buying n and (n – 1) units of the good.

**Note:**

1) An individual does not buy the good if it costs more than his reservation price for that good, but is eager to do so if it costs less.
2) If the price of a good is just equal to an individual's willingness to pay, he is indifferent between buying and not buying.

Now, consider Table 3.2 showing five potential buyers of commodity X, listed in order of their willingness to pay. At one extreme is individual A, who is willing to pay a price of Rs. 59 for a unit of commodity X. Individual B is less willing to have a unit of this good, and will buy one only if the price is Rs. 45 or less. C is willing to pay only Rs. 35, D only Rs. 25, and E, with the least willingness for the good, will buy one only if it costs not more than Rs. 10. How many of these five buyers will actually buy a unit of commodity X? It depends on the price. If the price of a unit is Rs. 55, only A buys one unit of commodity X; if the price is Rs. 40, A and B both buy a unit of commodity X, and so on. So the information in the table on willingness to pay also defines the *demand schedule* for commodity X. As we saw in Unit 1, we can use this demand schedule to derive the market demand curve shown in Fig. 3.1. Here we are considering only a small number of buyers, hence this curve doesn't look like the smooth demand curve you are familiar with, where markets contain hundreds or thousands of buyers. This demand curve is step-shaped, with alternating horizontal and vertical segments. Each horizontal segment, *i.e.* each step corresponds to one potential buyer's willingness to pay. However, we will see further in the unit that for the analysis of consumer's surplus it doesn't matter whether the demand curve is stepped or whether there are many consumers, making the curve smooth.

**Table 3.2: Potential buyers and their Willingness to Pay**

| Potential Buyers | Willingness to Pay |
|---|---|
| A | 59 |
| B | 45 |
| C | 35 |
| D | 25 |
| E | 10 |

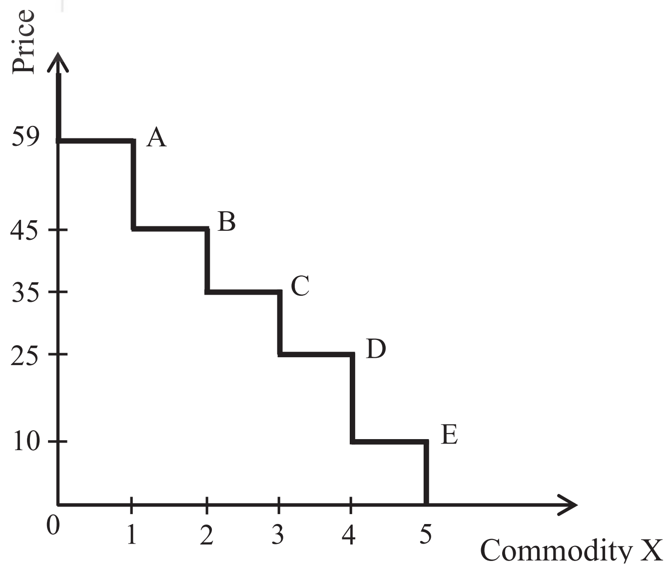


**Fig. 3.1: Demand function for discrete commodity X**

Now, suppose market price of commodity X is Rs. 30 per unit. At this price, individual A, B and C will buy the commodity. Do they gain from their purchases, and if so, how much? Individual A would have been willing to pay Rs. 59, so his net gain is Rs. 29 (= 59 – 30). Such a gain is possible due to the fact that the willingness to pay of the individual for one unit of the good is greater than the market price he actually paid for it. Similarly, individual B's net gain is Rs. 15 (= 45 – 30); C's net gain is Rs. 5 (= 35 – 30). Individual D and E, however, won't be willing to buy commodity X at a price of Rs. 30, so they neither gain nor lose. The net gain that a buyer achieves from the purchase of a good is called that buyer's *individual consumer's surplus.* The sum of the individual consumer's surpluses achieved by all the buyers of a good is known as the *total consumer's surplus* achieved in the market. In Fig. 3.2, the total consumer's surplus is the sum of the individual consumer's surpluses achieved by A, B, and C: Rs. 49 (=29 +15 +5), shown by the shaded area.

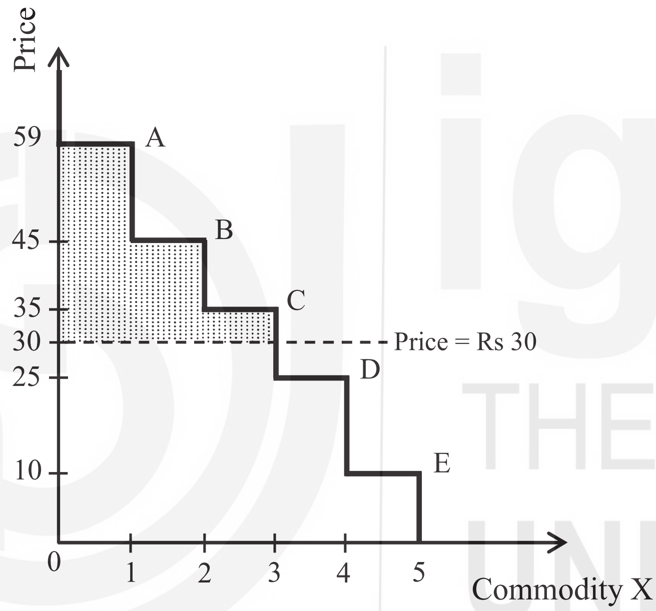


**Fig. 3.2: Consumer's Surplus when market price is Rs 30**

Another way to say this is that total consumer's surplus is equal to the area under the demand curve but above the price. The same principle applies regardless of the number of consumers. That is, when there are many potential buyers in the market for a commodity, then the demand curve is smooth without steps. The consumer's surplus in such a case is given by the same principle (the area below the demand curve but above the price). (Refer Fig. 3.3)

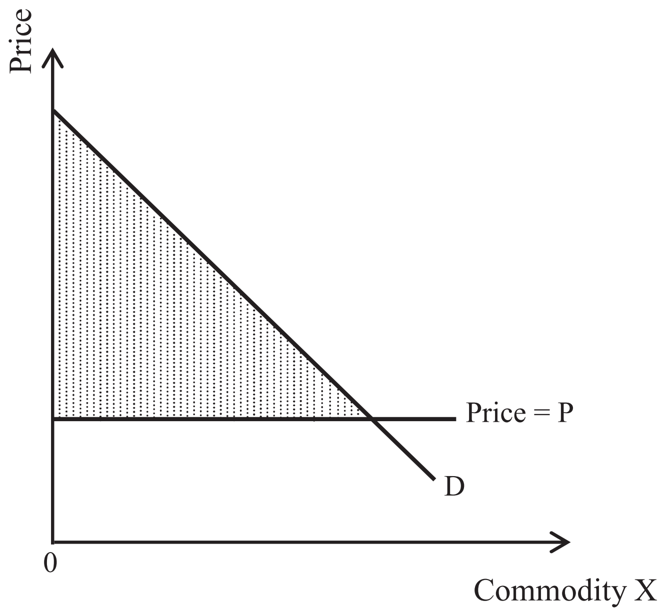


**Fig. 3.3: Consumer's Surplus with many potential buyers in the market**

### 3.3.2 Consumer's Surplus for a Non-discrete Good

Consumer's surplus for a non-discrete good is the area under the demand curve above the price line, found using a definite integral. Refer Fig. 3.4.

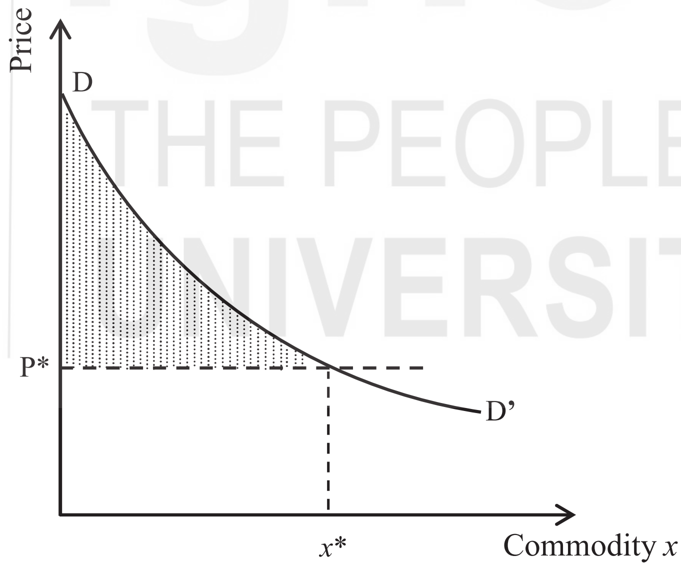


**Fig. 3.4: Consumer's Surplus for Non-discrete good**

Here, P* is the market price; x* the quantity demanded at P*. Consumer's surplus in this case also will be given by the area below the demand curve and above the market price line. The formula for calculation of consumer's surplus in case of continuous commodity involves the application of integration technique you have learnt in Unit 14 of your course on Mathematical Methods in Economics during first semester (BECC-102).

Consumer's Surplus = $\int_0^{X*}$(Inverse demand function $-$ P*) dx.

In other words,

$$\int_{\text{x coordinate of left edge}}^{\text{x coordinate of right edge}} [(\text{Upper function}) - (\text{Lower function})]$$

You will be able to understand better with an illustration below:

**Example 1:** Let us calculate the consumer's surplus for the following demand function given in Fig. 3.5.

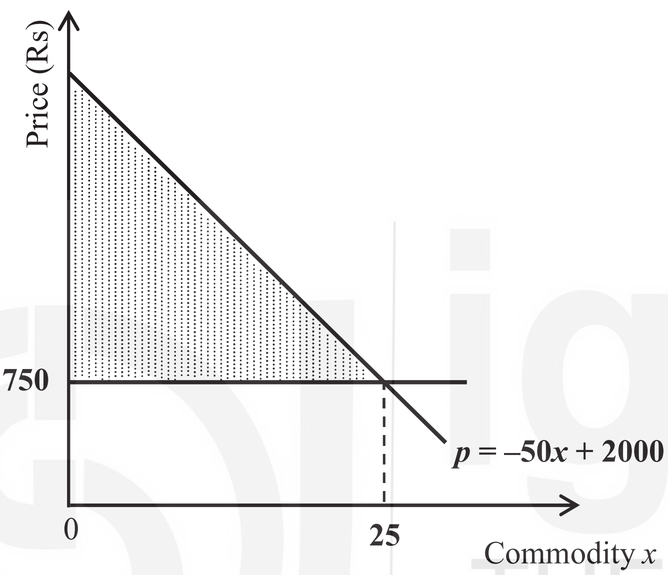


**Fig. 3.5: Consumer's Surplus for Non-discrete good**

**Solution:**

Look again at the shaded area for Consumer's Surplus. The left edge of the triangle has an *x*-coordinate of 0, and the right edge is our equilibrium point, which has an *x*-coordinate of 25.

The top of the triangle is the inverse demand equation p = – 50x + 2000, and the base of the triangle is our constant equilibrium price, Rs. 750. So,

Consumer's Surplus = $\int_0^{25}[(-50x + 2000) - (750)]dx$

= $[-25x^2 + 2000x - 750x]|_0^{25}$

= $[-25(25)^2 + 2000(25) - 750(25)] - [-25(0)^2 + 2000(0) - 750(0)]$

= 15,625 units

**Check Your Progress 1**

1) What is consumer's surplus, and how is it measured?

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2) Consider an individual who participates in the market as the buyer of shirts. His willingness to pay for subsequent units of Shirts is presented in the following Table 3.3 below:

**Table 3.3: Willingness to pay for units of Shirts**

| Units of Shirts | Willingness to Pay |
|---|---|
| $1^{st}$ unit | 60 |
| $2^{nd}$ unit | 50 |
| $3^{rd}$ unit | 40 |
| $4^{th}$ unit | 30 |
| $5^{th}$ unit | 20 |
| $6^{th}$ unit | 10 |

i) Use this information to construct individual's demand curve for shirts.

ii) If the price of a shirt is Rs. 20, how many units of shirts will the individual buy?

iii) Find consumer's surplus if the market price of a shirt is Rs. 20?

iv) If the price of a shirt rises to Rs. 40, how many units would he purchase?

3) Consider an individual demand curve given by the following equation, $q = -0.5p + 70$. Find the consumer's surplus for this individual when market price faced by him is Rs.100 per unit. (Note: In the question, the given equation is different from the one we considered in Example 1. There we encountered an inverse demand function. So the right approach will be to first convert the given equation into an inverse demand function and then proceed).

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## 3.4 CHANGE IN CONSUMER'S SURPLUS

Consumer's surplus, as we know by now, is determined by the willingness to pay for a good, given by the demand curve and the market price of that good. Thus, change in any one or both these determinants will have an impact on the consumer's surplus. Moreover, all the factors affecting the demand for a commodity by a potential buyer, like price of the commodity, price elasticity of demand, income, etc. will, in turn, affect the consumer's surplus too.

### 3.4.1 Effect of Price Changes on Consumer's Surplus

It is often important to know how much the consumer's surplus *changes* when the price changes. There exists an inverse relationship between consumer's surplus and the price.

Given the demand curve, *i.e.* willingness to pay by a potential buyer, a rise in the market price of the commodity reduces consumer's surplus, whereas a fall in the market price of the commodity, increases consumer's surplus.

The same approach we have used to derive consumer's surplus can be used to illustrate the above statement. Let us return to the example of the market for commodity X. Suppose that the market price fell from Rs. 30 to Rs. 20 owing too excess supply. How much would this increase consumer's surplus? The answer is illustrated in Fig. 3.6. As shown in the figure, there are two parts to the increase in consumer's surplus. Part I is the gain to those who would have bought commodity X even at Rs. 30. After the price fall, A, B and C, who would have bought X at Rs. 30, each gains Rs. 10 in consumer's surplus from the fall in price to Rs. 20. Part II is the gain to those who would not have bought X at Rs. 30 but are willing to pay more than Rs. 20. In this case comes the individual D, who would not have bought X at Rs. 30 but now does buy one at Rs. 20, gaining Rs. 5 (= 25 – 20). The total increase in consumer's surplus is Rs. 35 (Sum of the areas of Part I and II). Likewise, a rise in market price from Rs. 20 back to Rs. 30 would decrease consumer's surplus by an amount equal to the sum of areas of Part I and II.

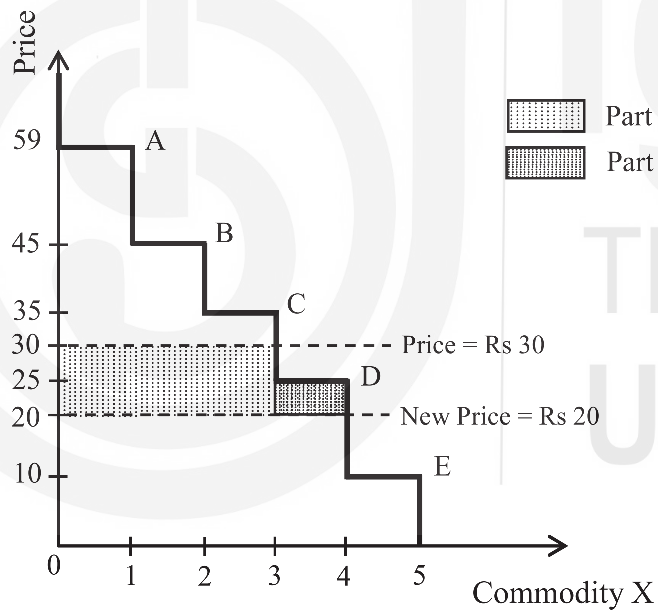


**Fig. 3.6: Change in Consumer's Surplus resulting from a Price change for discrete good**

Fig. 3.6 illustrates that when the price of a good falls, the total consumer's surplus (i.e., the area under the demand curve but above the price) increases. Fig. 3.7 shows the same result for the case of a smooth demand curve. Here we assume that the price of commodity X falls from P to $P_1$, leading to an increase in the quantity demanded from Q to $Q_1$ units and an increase in consumer's surplus (as given by the sum of the shaded areas).

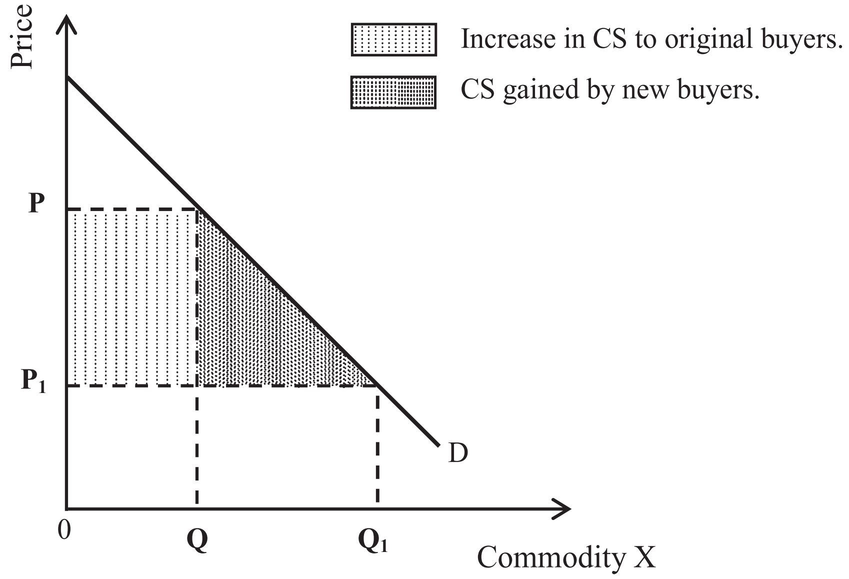


**Fig. 3.7: Change in Consumer's Surplus resulting from a Price change for Non-discrete good**

**Example 2:** When demand is estimated to be p = 6 – 0.5x, calculate the loss in consumer's surplus when a tax drives price from Re. 1 to Rs. 2.

**Solution:**

In order to calculate the loss in the consumer's surplus resulting from a price rise, first we plot the demand curve. The demand curve will be linear and downward sloping as presented in Fig. 3.8. The position of the demand curve will be determined by its intercepts with the price (p) and the quantity (x) axis.

When p = 6, consumer buys no amount of commodity x. Hence, the vertical intercept is at p = 6. When p falls to 0, our consumer is willing to consume not more than 12 units of commodity x. So x = 12 is the horizontal intercept. The demand curve as a straight line must pass through these intercept points.

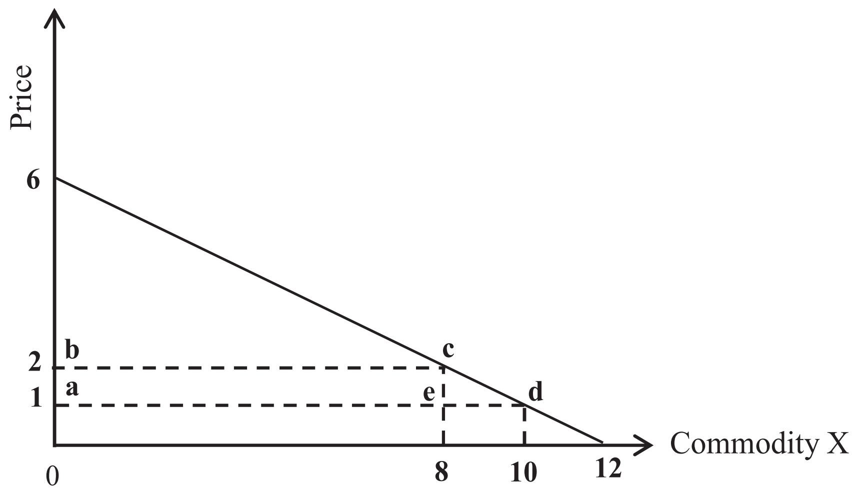


**Fig. 3.8: Change in Consumer's Surplus resulting from a Price change**

When p = 1, x = 10 and when p = 2, x = 8.

Loss in consumer's surplus = Area abcd

= Area abce + Area ecd

$= (8 \times 1) + \frac{1}{2} \times (10 - 8) \times 1$

= 8 + 1 = Rs. 9

**Check Your Progress 2**

1) Other things remaining equal, assuming demand relation holds for an individual and is observable, what happens to consumer's surplus if the price of a good rises? Illustrate using a demand curve.

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2) Consider a market for good X represented by the following demand function, q = 125 – 25 p. Now assume the market price of this good to be Rs. 3, calculate

a) The initial consumer's surplus at market price of Rs. 3.

b) The change in consumer's surplus when price falls to Re 1.

c) The gain due to fall in price to the consumers who could buy at old price of Rs. 3 (that is, the gain to the old buyers); also the gain to the new buyers of good X at lower price of Re 1.

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3) Assume that a market demand curve of a commodity is given as $q = 20 - p^2$. What will be the change in Consumer's surplus when market price of that commodity will rise from Rs. 2 to Rs. 3?

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### 3.4.2 Quasi-linear Preferences and Change in Consumer's Surplus

We have already learnt the concept of Quasi-linear preferences in Sub-section 2.9.3 of Unit 2. In practice, to compute the change in consumer's surplus, we need to first have an estimate of the consumer's demand function. Besides demand curve analysis, there also exists an indifference analysis approach to measure change in consumer's surplus. It depends upon some specific assumptions about the preferences of the consumer.

Now, let us bring into picture quasi-linear preferences and the underlying relation between change in consumer's surplus and change in utility. They both coincide when utility function is quasi-linear. Such a utility function is given by

$$U(x_1, x_2, \ldots x_k, y) = y + v\,(x_1,..x_k);\ v'(.) > 0,\ v''(.) < 0 \qquad (1)$$

Following is the specific form of quasi linear utility function:

$$U(x, y) = y + 5 \log x$$

Function (1) is linear in one good, here y, but non-linear in the other goods (here x). For this reason, this is called a quasi-linear utility function. For our two goods analysis, we will consider k =1 in the utility function (1) to get,

$$U(x, y) = y + v\,(x)\ ;\ v'(.) > 0,\ v''(.) < 0$$

U(x) is assumed to be a strictly concave function, an assumption which would imply that indifference curves will be strictly convex. *In utility functions of this form, utility is measured in the same units as the y-good, that is, every unit increase in y is a unit increase in utility; and if an increase or decrease in x yields a change in utility of $\Delta U$, the same change in utility could have been brought about by instead changing y by the amount $\Delta U$.*

Indifference curves of a quasi-linear utility function are parallel shifts of each other as shown in the following Fig. 3.9.

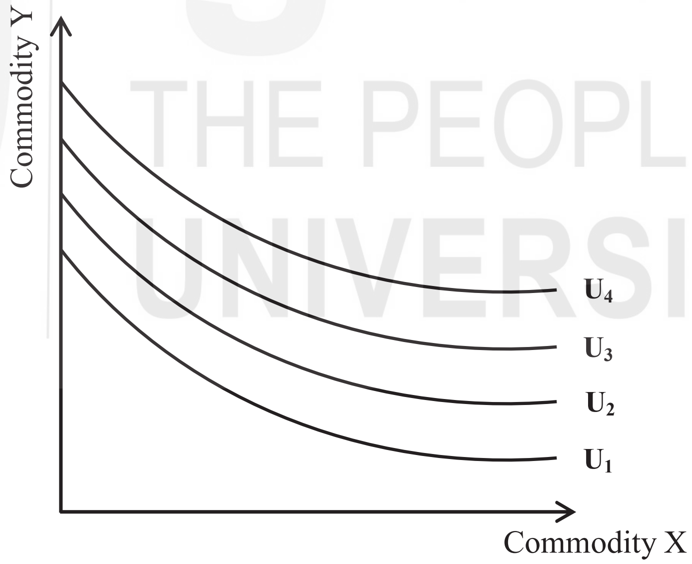


**Fig. 3.9: Quasi-linear Indifference Curves**

In case of Quasi-linear utility functions, Consumer's surplus is equal to the gain in utility. Let us consider an example to prove this statement. Consider a consumer with the utility function given by $U(x, y) = y + 12x - \frac{1}{2}x^2$, having a demand function x = 12 – p for good x, where we have assumed price of good y to be Re 1, price of good x to be Rs. p per unit, and income of the consumer to be Rs. 100. Suppose, consumer opts for a bundle (x = 0, y = 100), then his utility, given by $U = 100 + 12(0) - \frac{1}{2}(0)^2 = 100$. If he then purchases x = 8 at a price of p = Rs. 4, leaving him with y = 68, his utility will be U = 68 + (12) (8) − $\left(\frac{1}{2}\right)$ 64 = 132, a gain in utility of 32 units, or Rs. 32 worth of utility. One can easily check that his consumer's surplus, estimated as the area under the demand curve but the above the price line is also Rs. 32.

## 3.5 COMPENSATING AND EQUIVALENT VARIATIONS

Alfred Marshall definition of consumer's surplus assumed that the marginal utility of money is constant. Such an assumption was refuted by John Hicks. He proposed two methods as 'willingness to pay measures' to allow for monetary measurement of changes in utility. According to him, if consumer's preferences are known, it is possible to estimate the effect of variation in the prices of the goods on the consumer's utility in monetary terms by using two alternative measures called Compensating and Equivalent variation.

The compensating variation measure of a change in utility of a price variation is concerned with asking— How much do we have to increase or decrease consumer's income to completely offset the effect of a price variation on his utility? In other words, what change in consumer's money income will ensure that his utility remains same even after the price change? Here, consumer is compensated for the price change and hence the measure is called the *compensating variation (CV).* Thus, compensating variation refers to the amount of additional money a consumer is needed to be compensated, to reach their initial utility, after a change in prices. CV reflects new prices and the old utility level.

The equivalent variation measure on the other hand asks— How much increase or decrease in consumer's income will cause him same utility loss as will be caused by a price variation? Equivalent variation (EV) is a closely related measure that uses old prices and the new utility level. The equivalent variation is the change in wealth (or income), at current prices, that would have the same effect on consumer welfare as would the change in prices, with income unchanged. Both, compensating variation and equivalent variation answer the same question— how much of an income change is required to offset a price change, so that a consumer's utility is unchanged?

**Note:**

1) Compensating variation is the income change needed after price change to restore utility to pre-price change level.

2) Equivalent variation is the income change needed to bring utility to post-price change level.

3) Both of these variations are observable only when demand functions are observable and they satisfy the conditions implied by utility maximisation.

### 3.5.1 Indifference Curve Analysis

Indifference curves can be used to analyse the effect of a price rise on consumer's utility. Consider again a hypothetical situation where an individual chooses between good X and good Y in the following Fig. 3.10. Here, we are considering good Y to be a numeraire good with price of Re 1 per unit. Let the original price of good X be $p_1$ and income of the individual be M. The budget constraint of this individual at the original prices is $ML_1$ and has a slope of $-p_1$. The equilibrium is attained at the tangency of the utility function and the budget constraint given by point A.

Now, suppose price of good X rises to $p_2$ . Individual's new budget constraint becomes $ML_2$, with a slope of $-p_2$. After the price increase, individual's new equilibrium is given by point C. Clearly, this individual is worse off because of the price rise, as evident from the fact that his new equilibrium bundle choice (C) lies on a lower indifference curve $I_2$ giving a corresponding utility level $U_2$, instead of $I_1$ with a utility level $U_1$ (where, $U_1 > U_2$).

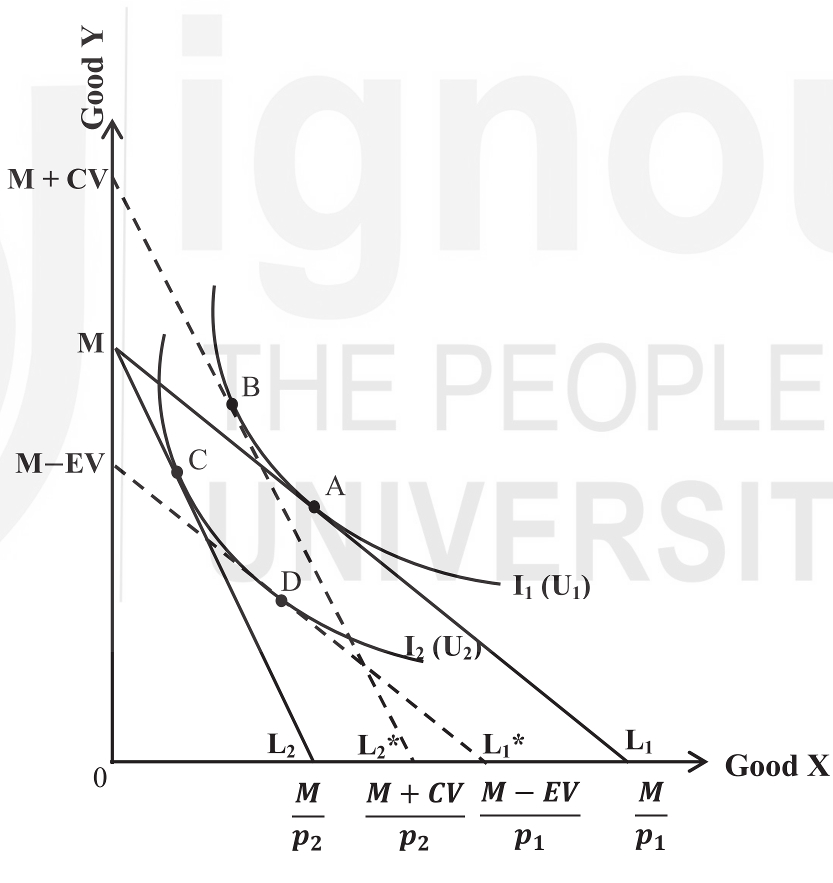


**Fig. 3.10: Compensation and Equivalent variations**

As discussed before, compensating variation (CV) is given by the amount of money that would fully compensate this individual for a price increase. Considering the above figure, as the price of good X increases from $p_1$ to $p_2$, individual is given enough extra income (that is CV) to bring the budget line back up to the old indifference curve ($I_1$), so that his utility remains at $U_1$. At the new income of M + CV, individual's budget line becomes $L_2$*, having the same slope, $-p_2$, as $L_2$. Under such conditions — that are of the price rise and the income compensation — individual would buy bundle represented by point B. Since we have assumed good Y to be numeraire, the monetary measure of the compensating variation will be equal to the difference between the two intercept values of good Y given by the Budget line, $L_2$ (*i.e.*, M) and the after-compensation Budget line, $L_2$* (*i.e.*, M + CV).

Equivalent Variation will be given by the amount of income that if taken from the individual, would lower his utility by the same amount as the price increase would have done. In Fig. 3.10, with price of good X remaining same at $p_1$, the harm that will be caused by the increase in price of good X to $p_2$ will have to be caused by an income fall. For this, individual's income would have to fall by enough to shift the original budget constraint, $L_1$, down to $L_1^*$, where it is tangent to $I_2$ at Bundle D. Since, we have a numeraire good on the vertical axis, equivalent variation will be given by the distance between the intercept of $L_1$ and that of $L_1^*$ on the axis representing good Y. The key distinction between these two measures of utility change, is that the equivalent variation is calculated using the new (lower) utility level, whereas the compensating variation is based on the original utility level.

### 3.5.2 Relation between Consumer's Surplus, Compensating Variation and Equivalent Variation

In general, Compensating Variation, Equivalent Variation, and Change in Consumer's Surplus ($\Delta$ CS) are not the same. But they do coincide if utility is quasi-linear. Here, we can easily make out that compensating and equivalent variations will be equal due to the fact that the resulting indifference curves are parallel. Our good Y being a numeraire good will ensure that the change in utility (equal to the compensating and equivalent variation measure), will in turn be equal to the change in consumer's surplus, as we observed in Sub-section 3.4.2. Now, let us look into the proof of this statement.

Consider a Quasi-linear utility function given by, U (x, y) = v (x) + y, where x and y are the two goods of consumption. Now, assume income of the consumer to be Rs. M, price of good x to be Rs. p and that of good *y* to be Re. 1 (*i.e.*, good y is assumed to be a numeraire good). If a consumer consumes $x_1$ units of good x, his consumption of good y will be $M - px_1$.

Now, Indirect Utility Function* will be, $V(p_x, p_y, M) = v(x_1) + M - px_1$

* We have seen in Unit 2 that the constraint optimisation of utility function U(x, y), subject to the budget constraint yields the demand functions $x(p_x, p_y, M)$ and $y(p_x, p_y, M)$. Plugging these two optimal values in the Utility function we get $U[x(p_x, p_y, M), y(p_x, p_y, M)] = V(p_x, p_y, M)$. *This function is called Indirect Utility function.* Thus, an *indirect utility function* gives the maximal utility the consumer can reach given the prices and income. It is obtained by substituting the utility maximising levels of goods x and y, given the set of prices, $p_x$, $p_y$, and the income, M, into the utility function.

When price of good *x* increases from p to p′, this alters the consumption of good x from $x_1$ to $x_1'$, which in turn changes the total utility from V to V ′ given by,

$$V' = v(x_1') + M - p' x_1'$$

We know that Compensating Variation (CV) is the amount of money satisfying the equation below:

$$\underbrace{v(x_1) + M - px_1}_{\text{before change}} = \underbrace{v(x_1') + (M + CV) - p'x_1'}_{\text{after change with compensation}}$$

$$\Longrightarrow CV = v(x_1) - v(x_1') + p'x_1' - px_1$$

Similarly, Equivalent Variation (EV) is the amount of money satisfying the equation below:

$$\underbrace{v(x_1) + (M - EV) - px_1}_{\text{before change with EV}} = \underbrace{v(x_1') + M - p'x_1'}_{\text{after change}}$$

$$\Longrightarrow EV = v(x_1) - v(x_1') + p'x_1' - px_1$$

Hence, we get CV = EV for the quasi-linear preference.

Now, Change in Consumer's Surplus (Δ CS) will be given by,

$$\Delta\, CS = \underbrace{[v(x_1) + M - px_1]}_{\text{Utility before change}} - \underbrace{[v(x_1') + M - p'x_1']}_{\text{Utility after change}}$$

$$\Longrightarrow \Delta CS = v(x_1) - v(x_1') + p'x_1' - px_1$$

Thus, we get ΔCS = CV = EV.

The above result can also be explained with the help of the Fig. 3.11. As we know that, in case of quasi-linear preferences, the indifference curves are parallel. Parallel indifference curves imply that the Marginal Rate of Substitution (MRS), that is, the slope of the tangent at a particular quantity of good *X* is the same on every indifference curve. This further would imply that the distance on the y-axis between the tangents to any two indifference curves at a particular value of good X must be equal to the constant vertical distance between them (which is U1 – U2 in terms of the value of the utility function).

Also, because the consumer's indifference curves are parallel, they will always require the same amount of good Y, U1– U2 , to move back to their original indifference curve (the compensating variation), regardless of the values of price of good X before and after the price change which changed their optimal bundle (and therefore utility level) (**Note:** With non-quasi-linear preferences, this amount will change depending on the precise value of price of good X, as we see in Fig. 3.10 illustrating the general case of non-equality between the CV and EV). This value, U1 — U2 is also the amount of cash income that the consumer would be willing to pay to avoid having the change in the price of good X in the first place (the equivalent variation).

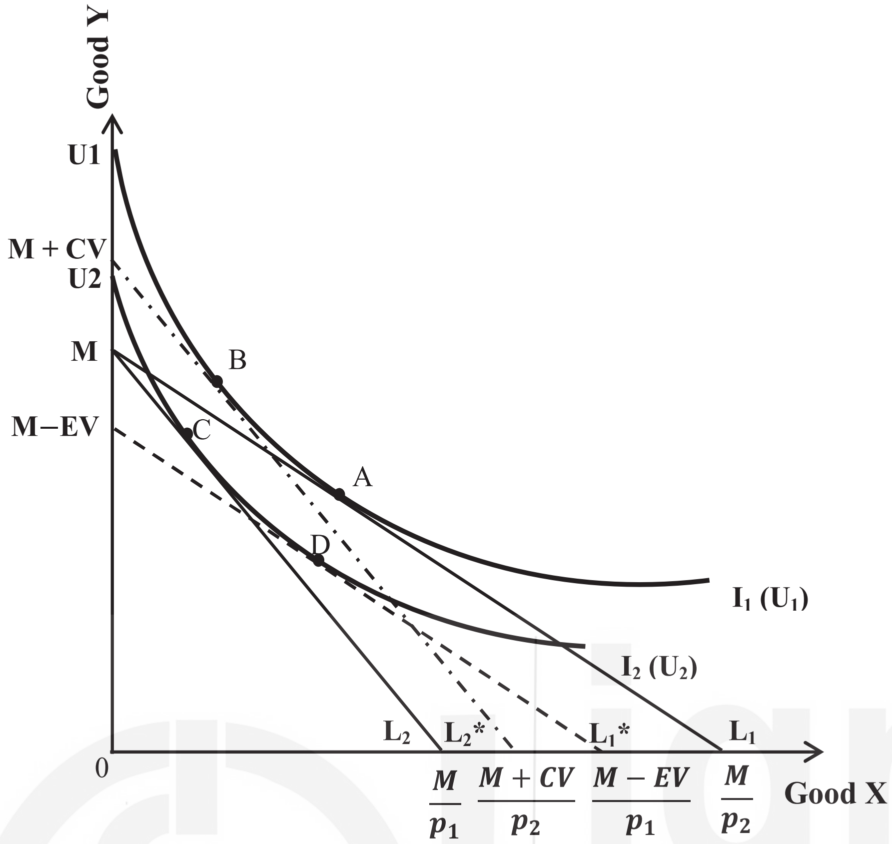


**Fig. 3.11: Compensation and Equivalent variations and change in consumer's surplus**

As per Fig. 3.11, EV = CV = U1 – U2. This holds only in case of quasi-linear preferences.

It can also be shown (though using techniques outside the scope of this unit) that in general when the preferences are not quasi-linear, the change in the surplus is between the compensating and equivalent variations.

**Example 3:** Consider an individual buyer with a quasi-linear utility preference over two goods X and Y, given by the relation $U(X, Y) = 10(X)^{1/2} + Y$. Here, good *Y* is the numeraire good with price ($p_y$) as Re 1. Let price of good X ($p_x$) be Re 1, which further rises to Rs. 2. Assuming individual's income to be Rs. 200, ascertain the two measures of utility change, that is, Compensating and Equivalent variation of a price change.

**Solution:**

We will start with solving for the equilibrium level of consumption of good X and good Y. That is,

The consumer will maximize U(X, Y) subject to the constraint, $p_xX + p_yY = M$.

For maximisation, the resultant lagrangean equation is given by

$$L = 10(X)^{1/2} + Y + \lambda\,[200 - (1)X - (1)Y]$$

Differentiating L with respect to X, Y, and $\lambda$, setting the derivatives equal to zero, and solving the resultant equations, we get the following optimal values of X and Y:

$X^* = 25$ ; $Y^* = 175$

After the price increase, the optimal bundle of consumption become:

$X^{**} = \frac{25}{4}$ ; $Y^{**} = 187.5$

In case of Quasi-linear utility functions, change in consumer's surplus is equal to the change in utility with different bundles of X and Y before and after the price change.

Thus, Change in Consumer's Surplus = U (25, 175) – U ($\frac{25}{4}$, 175)

$$= 10(25)^{\frac{1}{2}} + 175 - 10(\frac{25}{4})^{\frac{1}{2}} - 187.5$$

$$= 12.5$$

For computing Compensating Variation (CV), we will consider the following relation:

U (25, 175) == U ($\frac{25}{4}$, 187.5 + CV)

$$10(25)^{\frac{1}{2}} + 175 = 10(\frac{25}{4})^{\frac{1}{2}} + 187.5 + \mathrm{CV}$$

CV == 12.5

Now, for computing Equivalent Variation (EV), we will consider the following relation:

U ($\frac{25}{4}$, 187.5) == U (25, 175 – EV)

$$10(\frac{25}{4})^{\frac{1}{2}} + 187.5 = 10(25)^{\frac{1}{2}} + 175 - \mathrm{EV}$$

EV == 12.5

In this case, change in consumer's surplus equals compensating variation which equals equivalent variation.

**Check Your Progress 3**

1) What is meant by Quasi-linear preferences? What special feature does the indifference curve depicting such preferences possess?

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2) What are the two monetary measures of consumer utility change proposed by Hicks? Explain with the help of diagram the relationship between these measure and change in consumer's surplus in case of quasi-linear preference relation.

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3) Consider the preference relation of an individual for two goods x and y to be given by, U (x, y) = min[x, y]. Also assume income of this individual to be Rs. 12. Given the market price of good x ($p_x$) as Re 1 and that of good y ($p_y$) as Rs. 2, calculate the amount of compensating and equivalent variation for this individual when the price of good x increases to Rs. 2.

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## 3.6 LET US SUM UP

The concept of consumer's surplus is often used when economists are deciding how scarce resources should be employed. This is a measure of the benefits which accrue, above the costs, to the users of the resource in question. In the present unit, we discussed the consumption theory further in respect of the resultant gains to the consumer from market participation. We learnt about consumer's surplus, which was initially presented by Alfred Marshall as an economic tool to measure benefits and losses resulting from changes in market conditions. In case of observable demand curve, we observed and estimated the amount of consumer's surplus, be it for discrete or for non-discrete goods consumption, as the difference between the amount of money that a consumer actually pays to buy a certain quantity of a good or service, and the amount that he would be willing to pay for this quantity rather than do without it. We even calculated the extent of utility change in terms of change in the amount of consumer's surplus resulting from price change of the good or service consumed.

As Marshallian consumer's surplus concept assumes constant marginal utility of money, which may not be the case always, subsequently the concept was redefined by John Hicks using indifference analysis, inducing the use of compensating and equivalent variations in utility economics. It answered the very relevant question, how does consumer utility change when the price changes? Equivalent variation answered it in terms of the amount of money we would have to take away from the consumer before the price change to leave him just as well off as he would be after the price change. Whereas, compensating variation answered it in terms of the amount of extra money needed to give to the consumer after the price change to make him just as well off as he was before the price change. We also learnt about the quasi-linear preferences and that consumer's surplus is equal to the change in utility in case of such preferences. At the end, we touched upon the relationship existing between the change in consumer's surplus, compensating variation and equivalent variation, noticed with proof that the three measures of utility change are equal in case of quasi-linear utility preferences.

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## 3.9 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

**Check Your Progress 1**

1) Read Section 3.2 about specifying a set.

2) i) Read Section 3.3 and draw the demand curve; ii) 5 units of good X ; iii) At a price of Rs. 20, individual's consumer's Surplus would be Rs.100 ; iv) only 3 units of good X

3) Rs. 400

**Check Your Progress 2**

1) Consumer's surplus, as the area below the demand curve and above the price line will fall due to price rise. Refer sub-section 3.4.1 and explain with the help of a diagram.

2) i) Rs. 50

   ii) Rs. 150

   iii) CS gain to old buyers = Rs. 100 ; CS gain to new buyers Rs. 50.

(**Hint:** Plot the demand curve by keeping q = 0 for vertical intercept, and finding q at the price level of Rs. 3 and then at Re 1. You may also refer to Example 2 and proceed.)

3. $-\frac{41}{3}$

(**Hint:** Change in Consumer's surplus = $\int_3^2 (20 - p^2)\mathrm{dp} = 20\mathrm{p} - \frac{1}{3}\mathrm{p}^3 \Big|_3^2$

$$= 20 \times 2 - \frac{1}{3} \times 8 - \left(20 \times 3 - \frac{1}{3} \times 27\right) = -\frac{41}{3})]$$

**Check Your Progress 3**

1) Refer Sub-section 3.4.2 and answer. The indifference curves representing such preferences are parallel, or in other words are vertically shifted versions of one indifference curve.

2) The two monetary measures of consumer utility change proposed by Hicks are Compensating and Equivalent variation. Refer sub-section 3.5.2 to explain and answer the second part.

3) CV = 4 ; EV = 3

[**Hint:** Given the utility function, U(*x*, *y*) = min [x, y], indirect utility function V ($p_x$, $p_y$, M) will be given by, V ($p_x$, $p_y$, M) = $\frac{M}{P_x+P_y}$.

For CV, insert values in the relation, V ($p_x$, $p_y$, M) = V ($p_x'$, $p_y$, M + CV), where $p_x'$ = 2 (the increased price of good x).

For EV, insert values in the relation, V ($p_x$, $p_y$, M – EV) = V ($p_x'$, $p_y$, M)]

# UNIT 4 CHOICE UNDER UNCERTAINTY AND INTERTEMPORAL CHOICE

**Structure**



## 4.0 OBJECTIVES

After going through this unit, you will be able to:

- state the concept of uncertainty and risk;
- discuss the expected utility function and its properties;
- explain how to maximise utility under uncertainty;
- discuss the attitude of an individual towards Risk;
- elucidate how risk aversion is dealt with institution of insurance; and
- appreciate the process of the intertemporal decision-making by the consumer.

## 4.1 INTRODUCTION

We have learned in Unit 2 how a consumer decides which combination of goods and services to buy, given his income and prices of the goods, in order to maximise his satisfaction. For this, there is a pre-supposition that the consumer has complete and perfect information and knowledge about the transaction. However, in real life situations, there are many uncertainties that consumers have to face before they decide. Uncertainty is a fact of life. In decision-making process, there are many uncertainties and randomness that a consumer has to take into consideration. For example:

i) Used Car: When you buy a used car, you are very unsure about its condition. You might be lucky and get a beautifully maintained car with no mechanical problems, or you might get a lemon or damaged car (whose mechanical problems are not easily observable).

ii) College/University: Suppose you choose a university for an undergraduate degree. Your university is very expensive. During the time of making decision about your college/university choice, how much do you know about it (or them)? Do you know how many professors are interesting and informative, and how many are deadly dull and uninterested in teaching? Do you know what your major will be? Do you know what a blessings or a curse your roommates or classmates might be?

iii) Life Insurance and Annuities: If all the events and contingencies are well known in life, then there is no need for insurance in such a society. However, when significant uncertainties are present, insurances solve the problem for such a society. You fear you may die too young, and you want to buy a life insurance policy to protect your spouse and children in the vent of your premature death. What kind of policy should you buy, and how much insurance should you have? Alternatively, you think you may live too long and you may run out of savings before you go. You do not want to be a burden to your children. You heard that you can buy an insurance against this possibility also. Should you buy an annuity, and how big an annuity should you get?

iv) Investments: You have some money that you are going to invest in bank term deposits (with minimal risks and also minimal rewards), or in shares of stocks (with considerable risks but greater rewards). What should you do?

v) Dangerous activities: You travel between home and college by car, or by train. Be it any mode, each time you face some risk of a fatal accident. Do you know what the odds of a road accident are? Suppose you own a car and are good at driving. But you do not know whether everyone around on the road is a careful driver. Someone may hit your car and the damages can be huge and varied. Hence, whenever you drive out from your house, you are uncertain that you will come back without getting hit or what will be the amount of accumulated damage in any accident.

There are countless examples in real life which involve uncertainty. Many of the things we do increase uncertainty in our lives, whereas other things reduce it. Sometimes we pay money to buy risks (like gamble, lottery) at other times we pay money to avoid risks. Under such uncertainties, there often exist difficulties in decision-making. To tackle this, institutions exist. Insurance in an economy is another such institution which helps in mitigating risks that arise due to uncertainties. With uncertainties, you might like to get insurance cover against car accidents. For this reason agents buy various kinds of insurance (life insurance, car insurance, fire insurance, crop insurance etc), to cover up for the risk involved or an unforeseen contingency.

In this unit we shall explain the notion of uncertainty and how it impacts the consumer's behaviour. The concept of expected utility function will be introduced in order to understand consumer's decision-making under uncertainty. In this connection, we will explain the concept of risk and an individual's attitude towards risk and the basic principle of choosing insurance.

We shall also throw light on intertemporal decision making, where Inter-temporal means across the time period. We shall take into consideration the saving and borrowing by a consumer and how does they affect his/her decision-making. This we will attempt to discuss with the help of an inter-temporal budget constraint.

## 4.2 REPRESENTATION OF UNCERTAINTY— PROBABILITY DISTRIBUTION

How does uncertainty affect consumer's decision-making? In the presence of uncertainty we have what is called— *probability* of occurrence of an event. For example, if probability that a consumer's income will increase is 90 per cent, then there is 10 per cent chance that his income might stay the same. If there is complete information about the probability of occurrence of an event, we can construct what is called a *probability distribution*. A probability distribution shows the likelihood that a given random variable will take up any of the given values. For example, the random variable in our example is the individual's salary hike. Table 4.1 shows probability distribution of individual's salary hike, that is it shows the likelihood that income hike of the individual will take various values.

**Table 4.1: Probability distribution of an Individual's Salary Hike**

| Salary Hike (Rs.) | Probability |
|---|---|
| 0 (No change) | 0.1 |
| 1000 | 0.1 |
| 3000 | 0.1 |
| 5000 | 0.4 |
| 7000 | 0.2 |
| 9000 | 0.1 |

This probability distribution is represented below in the Fig. 4.1.


**Fig. 4.1: Discrete Probability Distribution**

In the above diagram, we see probabilities of occurrence on the y-axis and salary hike on the x-axis. Since there is finite number of events in this example, we call it discrete probability distribution. Given the above example, we can now say, probability that the salary hike will be Rs. 5000 is 40 per cent and that the hike will be Rs. 9000 the probability is only 10 per cent.

Next, let us calculate, expected salary hike given the information regarding the probability of their occurrence. So to calculate expected salary hike, we multiply each salary hike with its respective probabilities, so we get,

Expected Salary Hike = 0(0.1) + 1000 (0.1) + 3000 (0.1) +
5000 (0.4) + 7000 (0.2) + 9000 (0.1) = 4700

Therefore, expected salary hike is equal to Rs. 4700.

Hence the expected value is nothing but the mean or the weighted average of a random variable X. The data on random variable will be scattered around its mean, that is, around its expected value.

Symbolically, Expected Value (EV) = $\sum X_i P_i$

where $X_i$ is the random variable value at i and $P_i$ is the associated probability. Accordingly, we require $0 < P_i < 1$ and also $\sum P_i = 1$

In the example that we are discussing here, the number of events (salary hikes) is finite and the probability distribution is simply the vertical bars that we see in Fig. 4.1. If, however, the number of events (let say height in centimeters) were infinite, then the probability distribution would look like the smooth curve as in Fig. 4.2. Such a probability distribution is called a *continuous probability distribution.*

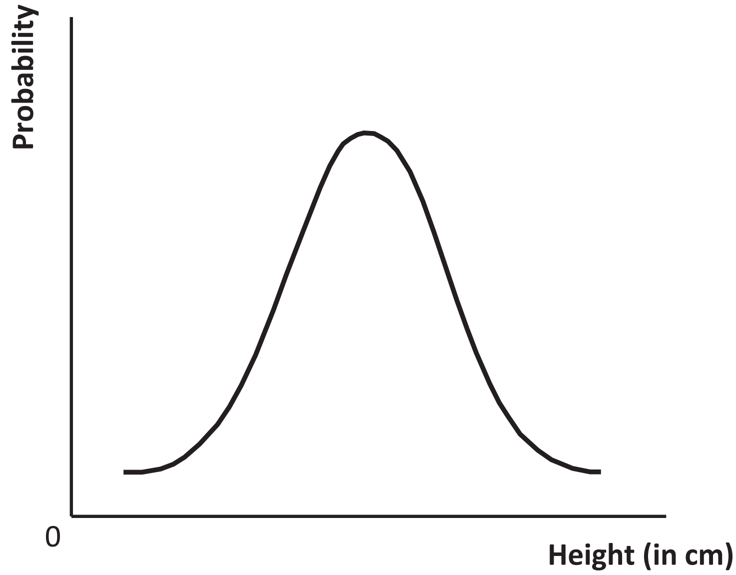


**Fig. 4.2: Continuous Probability Distribution**

## 4.3 DECISION-MAKING UNDER UNCERTAINTY

In the previous units, we discussed about the choices that the agents make under the conditions of certainty. When uncertainty is involved, altogether a different decision-making process will be followed. Consider an agent who is to make a choice between two possible investment projects offering different payoffs with respective probabilities. Please note here that the decision-maker does not know what would happen once he decides between the two investment plans, *i.e.* investment A or B. The outcome here depends upon some random factors which are out of control of the decision-maker. The random variable here will be the payoff or benefit to the decision-maker by investing in either plan. Consider Table 4.2, giving different payoffs from each plan along with their respective probabilities.

**Table 4.2: Probability distribution of payoffs from different investment Plans**

| Payoff from Plan A | Probability | Payoff from Plan B | Probability |
|---|---|---|---|
| 10 | 0.1 | 10 | 0.0 |
| 20 | 0.3 | 20 | 0.3 |
| 30 | 0.2 | 30 | 0.4 |
| 40 | 0.2 | 40 | 0.3 |
| 50 | 0.2 | 50 | 0.0 |

This is how the payoffs are read— there is 10 per cent chance of getting Rs. 10 payoff, 30 per cent chance of getting Rs. 20 payoff and 20 per cent chance each of getting payoff as Rs. 30, Rs. 40 and Rs. 50. Similarly for plan B, we have the outcomes (payoffs) and their respective probabilities. Now to choose between plan A or plan B, agent needs to calculate the expected payoff from each plan.

From Plan A, his expected payoff = $10(0.1) + 20(0.3) + 30(0.2) + 40(0.2) + 50(0.2) = 31.$

Similarly, from Plan B, expected payoff = 30.

As we might expect, our agent's first inclination is to choose the investment that provides the highest expected monetary value. This approach seems to make sense. Most people would want to consider the investment from which they can expect the greatest return. On such basis, our agent will choose plan A because its expected payoff value is greater than the expected payoff value of plan B. Hence when faced with uncertainties, it is natural to believe that the agents maximise their expected monetary benefits which, in turn, maximises their expected utility. But this is not true in all cases. Now let us discuss a situation in which maximising expected monetary value may render different result than maximising expected utility.

**Why Not Maximise Expected Monetary Returns?**

Although it seems logical to use expected monetary value as the criterion for making investment decisions under conditions of uncertainty, this approach is actually filled with contradictions. Consider the following example:

Let us say that a patient leaves a doctor's office with the sad news that he has exactly two days to live unless he is able to raise Rs. 20,000 for a heart operation. The patient spends the next two days calling relatives and friends but is not able to raise a penny. With one hour left to live, the patient walks dejectedly down the street and runs into a dealer. Instead of offering the patient Rs. 20,000 outright, this dealer offers him a choice between two gambles. In gamble A, he will receive Rs. 10,000 with a probability of 0.50 and Rs. 15,000 with a probability of 0.50. In gamble B, the patient will receive nothing (Rs. 0) with a probability of 0.99 and Rs. 20,000 with a probability of 0.01. These gambles are summarised in the following Table 4.3.

**Table 4.3: Probability distribution of payoffs from Gamble A and B**

| Gamble A | | | Gamble B | | |
|---|---|---|---|---|---|
| **Prize (Rs)** | **Probability** | **Utility of Rupee** | **Prize (Rs)** | **Probability** | **Utility of Rupee** |
| 10,000 | 0.5 | 0 | 0 | 0.99 | 0 |
| 15,000 | 0.5 | 0 | 20,000 | 0.01 | 1 |
| Expected monetary value = Rs. 12,500<br>Expected utility = 0 | | | Expected monetary value = Rs. 200<br>Expected utility = 0.01 | | |

Obviously, if our patient is a maximiser of expected monetary value, he would like to choose gamble A, with expected return of Rs. 12,500, whereas gamble B's expected return is only Rs. 200. However, there is a catch here. If our patient chooses gamble A, then it is certain that he will die in one hour, but if he chooses gamble B, he has at least a 1% chance to live. Hence, the money to be received by our patient in gamble A is worthless because he will die, while gamble B promises a chance to live. Therefore, most people would say that gamble B is the better choice. *The reason is obvious. Most people are interested in more than just obtaining an amount of money. They are also interested in what that money will bring in terms of happiness or satisfaction.* In this case, because Rs. 20,000 is needed for a lifesaving operation, any amount below Rs. 20,000 is worthless. Hence, if we arbitrarily call the value or utility of death 0 and the value or utility of living 1, we can see that from the patient's point of view, the expected utility of gamble A is 0 [0(0.5) + 0(0.5) = 0] and the expected utility of gamble B is 0.01 [0(0.99) +1 (0.01) = 0.01]. If people act, so as to maximise their expected utility, then gamble B is better than gamble A and it is the one that will be chosen by our patient.

The point of this example, then, is that when making decisions in situations involving uncertainty, agents do not simply choose the option that maximises their expected monetary payoff; they also evaluate the utility of each payoff. We might say that they behave as if they are assigning utility numbers to the payoffs and maximising the expected utility that these payoffs will bring.

### 4.3.1 The von Neumann-Morgenstern (vNM) Expected Utility Function

In their book 'The Theory of Games and Economic Behaviour', John von Neumann and Oskar Morgenstern developed mathematical models for examining the economic behaviour of individuals under conditions of uncertainty. Note that the new utility function is no longer be purely ordinal, it will have some properties of cardinal utility function; however it will be providing the basis for the rigorous analysis of choice under uncertainty. Consider a risky situation, where the decision-maker does not know beforehand which state of the world will occur. For simplicity we assume two possible state of the world situations — 1 and 2, with respective probabilities of their occurrence as $\pi_1$ and $\pi_2$. Let $c_1$ denote individual's consumption if state 1 occurs and $c_2$, if state 2. One of the convenient ways to represent von Neumann-Morgenstern expected utility function is in the following form:

$$u(c_1, c_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

Where, function v(.)* gives the amount of utility attained from some amount of consumption. Thus vNM expected utility can be written as a weighted sum of some function of consumption in each state, $v(c_1)$ and $v(c_2)$, where the weights are given by the probabilities $\pi_1$ and $\pi_2$ (where $0 < \pi_i < 1$ and also $\Sigma\pi_i = 1$ with i = 1, 2). If one of the states is certain, so that $\pi_1 = 1$ say, then $v(c_1)$ is the utility of certain consumption in state 1. Similarly, if $\pi_2 = 1$, $v(c_2)$ is the utility of certain consumption in state 2.

When we say that a consumer's preferences can be represented by an expected utility function, or that the consumer's preferences have the expected utility property, we mean that we can choose a utility function that has the additive form described above.

VNM utility theory is based on the following assumptions:

1) **Completeness and transitivity:** The consumer's preference over all the alternatives, certain and uncertain, are complete and transitive. It is an obvious extension of the assumption we made for the standard consumer theory model.

2) **Continuity:** Suppose consumer prefers alternative X over Y and Y over Z, that is Y is somewhere between X and Z in the consumer's preference ranking. Then there must exist a probability $p_x$, with $0 < p_x < 1$, such that the consumer is indifferent between the middle alternative Y and the lottery offering best alternative X with probability $p_x$ and the worst alternative Z with probability $(1 – p_x)$.

3) **Independence:** Suppose consumer is indifferent between alternatives X and Y, and Z is any other alternative. Consider two lotteries— one with outcomes X and Z and the other with Y and Z. Suppose both these lotteries assign the same probability to the indifferent alternatives (i.e., X or Y), and therefore the same probability to the other alternative Z. Then the consumer must be indifferent between these two lotteries.

4) **Unequal probabilities:** Suppose consumer prefers alternative X to Y. Consider two lotteries, both having only X and Y as possible outcomes, with both attaching different probabilities to the two outcomes; then consumer prefers the lottery that assigns a higher probability to her preferred outcome X. Thus a consumer would prefer the gamble that gives him/her better odds of the preferred prize.

5) **Compound lotteries:** A consumer is given a choice between lotteries. Lottery $L_1$ is the straightforward lottery that provides certain outcomes $(X_1, X_2, \ldots, X_n)$ with respective probabilities $(p_1, p_2, \ldots, p_n)$. Lottery $L_2$ is a lottery whose outcomes are other lotteries. However, after the intermediate lotteries play out, it ultimately ends up with a same certain outcomes, and with the same respective probabilities $(p_1, p_2, \ldots, p_n)$. Then the consumer is indifferent between $L_1$ and $L_2$. Thus a rational consumer would focus on the ultimate probabilities of the ultimate outcomes.

Thus Von Neumann Morgenstern (vNM) Expected Utility theorem says: Let L be the risky alternative, that is, any lottery. Suppose its outcomes are $(X_1, X_2, \ldots, X_n)$. These are certain outcomes or other lotteries with respective probabilities $(p_1, p_2, \ldots, p_n)$. Then the utility of the risky alternative L is the expectation of the utilities of its possible outcomes. That is:

$$U(L) = p_1 U(X_1) + p_2 U(X_2) + \cdots + p_n U(X_n)$$

**Check Your Progress 1**

1) Which of the following utility functions have the expected utility property?

   a) $u(c_1, c_2) = 5\,[\pi_1 v(c_1) + \pi_2 v(c_2)]$

   b) $u(c_1, c_2) = \pi_1 c_1 + \pi_2 c_2$

   c) $u(c_1, c_2) = \pi_1 \ln c_1 + \pi_2 \ln c_2 + 17$

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2) What is a probability distribution? How does it explain the choice under uncertainty?

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3) Consider an individual who invests in different schemes and faces an uncertain income flow of Rs 5,000, or Rs 8, 000, or Rs 10,000 with respective probabilities of 10%, 50%, and 35%. Determine the expected monetary value of the investment schemes.

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4) Explain von Neumann Morgenstern utility function.

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## 4.4 ATTITUDE TOWARDS RISK

How does an individual decide when facing risks? Assuming that the decision-making by such an individual is rational, his attitude towards risk can be ascertained. In this section we attempt to employ the concept of expected utility function to see how 'rational' decisions are made in the face of risk. We will define here, what is called— the risk in choice-making, which arises due to uncertainty; and how institution of Insurance helps to overcome such risks.

### 4.4.1 Risk

Risk is generally perceived as the possibility of facing a misfortune or a loss. In other words, it is the potential that a choice or a decision made will bring an undesirable outcome. Like your decision to drive a car on road involves the risk of your car meeting an accident and hence incurring damage or loss. That is here, risk is involved with uncertainty of happening or not happening of an event. Such uncertain situations are often connected to some associated probabilities of occurrence or non-occurrence of an event.

Now the question arises, "How do people react to events involving risk compared to those that are risk-free?" There exists heterogeneity in people's preference toward risk. Consider an individual, who is asked to choose between two gambles, G1 and G2. Gamble G1 offers a prize of Rs. 50 with certainty, whereas gamble G2, offers a prize of Rs. 100 with a probability of 0.50 and no prize with a probability of 0.50. G1 being a sure thing is obviously less risky than G2. Expected value of the gambles will be given as follows:

Expected Value (EV) of G1 = $1 \times 50$ = Rs. 50 (as probability of a certain payoff is 1.)

Expected Value (EV) of G2 = $0.50 \times 100 + 0.50 \times 0$ = Rs. 50

Although both Gambles have the same EV, one can only be sure about G1 as G2 is risky in Economic sense. While making a choice between G1 and G2, some people might be indifferent among G1 and G2; some might prefer G2, the risky one; and others might choose G1, the safe gamble. From this, emerges the notion of attitude towards risk. Suppose a consumer is given the choice between two activities, a risk-free activity with a guaranteed outcome and a risky activity involving outcomes with some probabilities. Assuming people prefer to maiximise utility attained from different available options and not associated monetary payoffs, they can be classified according to their attitude towards risk into three categories, *viz*— Risk Neutral, Risk Averse and Risk Loving. Let's discuss each one of these three categories.

### 4.4.2 Risk Neutrality

A risk neutral person shows no preference between a certain income, and an uncertain income, given they both have equal expected value. In other words, he will be indifferent between a risky and a risk-free choice, if both result in same expected value to him. In our example above of two gambles (G1 and G2), a risk neutral person would be indifferent between the two, as both have equal expected value (= Rs. 50).

Now we attempt to graphically present behaviour of such an individual. Consider an individual facing risky activity A that generates a payoff (X) of Rs. 100 with probability $\frac{1}{4}$ and a payoff of Rs. 1000 with probability $\frac{3}{4}$.

Expected Value (EV) of activity A will be given by $(\frac{1}{4} \times 100) + (\frac{3}{4} \times 1000) =$ Rs. 775.

Let utility function representing preferences of this individual be given by

U(X) = 2X, where X represents the payoff in Rupees.

Now, consider Fig. 4.3 where Payoffs (Rs) appear on the horizontal axis and the utility generated by those payoffs, on the vertical axis.

Utility attained when payoff X = 100 (represented by point a) is given by

U(100) = 2(100) = 200;

Similarly, when payoff X = 1000, utility attained (represented by point b) is given by U(1000) = 2(1000) = 2000.

Expected Utility [E(U)] of the risky activity A will given by

$[\frac{1}{4} \times U(100) + \frac{3}{4} \times U(1000)] \Rightarrow [\frac{1}{4} \times 200 + \frac{3}{4} \times 2000] = 1550$, this is nothing but the height at point c, representing mean value of the line joining points *a* and b.

Now, suppose a risk-free activity B is offered to the same individual, where he receives a certain payoff, which equals the EV (expected value) of the Risky activity A, *i.e.* Rs. 775. Then, EV of risk-free activity *B* will also be Rs. 775, as individual will be receiving the amount with certainty (i.e. probability = 1).

Utility attained from activity B [let us denote it by U(EV) as activity B's payoff is nothing but equal to the EV of activity *A*] will be given by

U(775) = 2(775) = 1550, which is again equal to the height at point c.

Now, points a (100, 200), b (1000, 2000) and c (775, 1550) can be joined to form a straight line, representing individual's utility function. Every point on this straight line curve tells us how much utility he will receive from any given level of rupees.

A straight line utility curve indicates that the individual's attitude towards risk will be neutral. This stems up from the fact that when faced with two activities A and B, with former being risky and the latter, risk-free, just because they both had same expected payoff value (= Rs. 775), the individual with the given utility function [U(X) = 2X] attained same utility from both the activities. That is,

Expected utility of risky activity A [E(U)] = Utility from risk-free activity B [U(EV) ] = 1550.

Note, however, that because the utility function is a straight line, every time the agent obtains one more Rupee, his utility increases by the same amount.

To put it another way, the marginal utility of an additional Rupee is constant, no matter how many Rupees the agent already has.



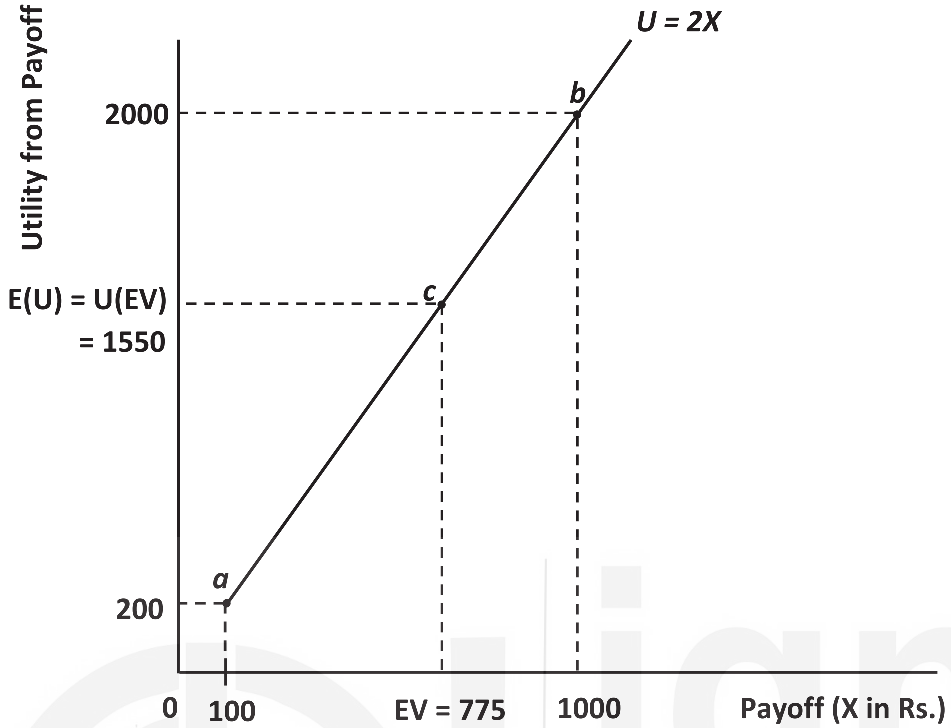


Fig. 4.3: Risk Neutrality

### 4.4.3 Risk Aversion

Now we consider an individual who has an aversion to risk, i.e. one who strongly dislikes risk. The utility function representing a risk-averse individual is not a straight line but rather is concave (refer Fig. 4.4). Concavity of the function implies that its slope is decreasing, which in turn implies that this individual exhibits diminishing marginal utility for additional units of payoff. In other words, the risk-averse consumer is not willing to incur additional risk for the possibility of a higher valued payoff. Unlike his risk neutral counterpart, such an individual will not be indifferent between a risk-free and a risky activity, each of which has the same expected monetary value.

To graphically understand such a behaviour, let us consider an individual who faces an option to either choose a risky activity A with a 20% (= $\frac{1}{5}$) chance of obtaining a payoff of Rs. 36 and a 80% (= $\frac{4}{5}$) chance of obtaining Rs. 100, or go for a risk-free activity B with a certain payoff equal to the expected payoff of the risky activity A, i.e., Rs. 87.2 $\left(= \frac{4}{5} \times 100 + \frac{1}{5} \times 36\right)$.

Now assume this individual has utility function given by U (X) = $\sqrt{X}$, where X is the payoff received. We will proceed in the similar way like we did in case of risk-neutral individual (Refer Fig. 4.4).

U(36) = $\sqrt{36}$ = 6, the height at point a; U(100) = $\sqrt{100}$ = 10, height at the point b.

E(U) of the risky activity A = $[\frac{1}{5} \times U(36) + \frac{4}{5} \times U(100)] \Rightarrow [\frac{1}{5} \times 6 + \frac{4}{5} \times 10] = 9.2$, height at point c, representing mean value of the line joining points a and b.

A risk-free activity B will offer a certain payoff of EV of the Risky activity A, *i.e.* Rs. 87.2. EV of risk-free activity B will also be Rs. 87.2.

Utility attained from activity B, i.e. [U(EV)] = U(87.2) = $\sqrt{87.2}$ = 9.338, which is equal to the height at point d. Here, we get U(EV) > E(U). That is, this individual attains higher utility from risk-free activity B, as compared to the expected utility from risky activity A (9.338 > 9.2). Such a behaviour exhibits an individual's risk averseness, where expected utility associated with a risky choice is less than the utility attained from a certain outcome (= the expected outcome of the risky choice) of a risk-free choice.

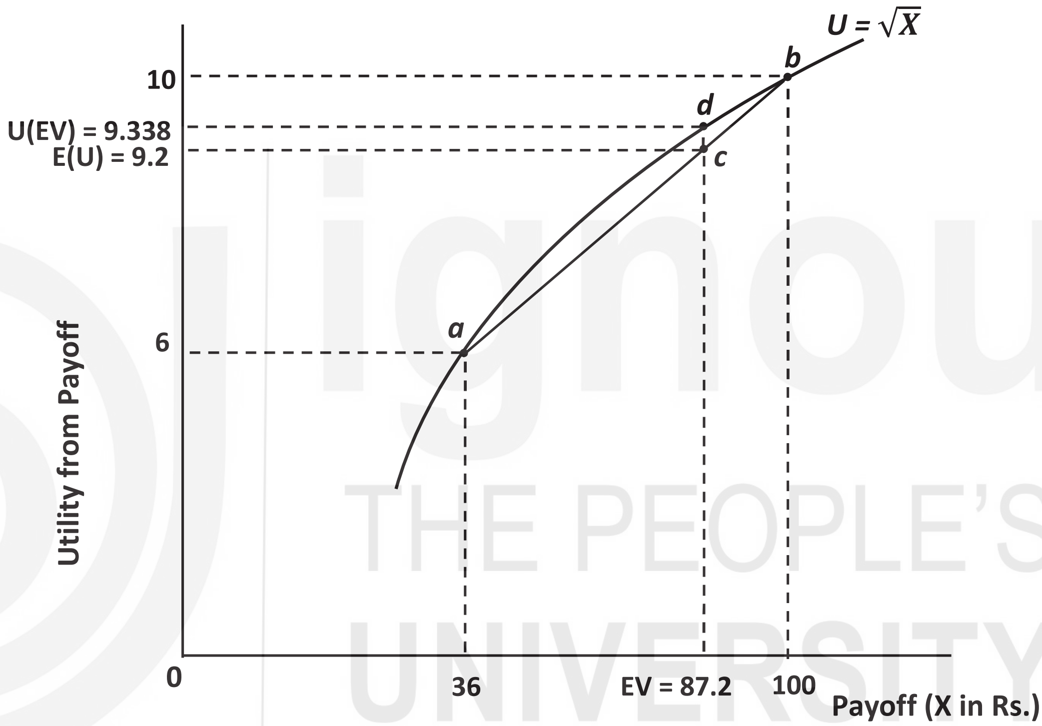


Fig. 4.4: Risk Aversion

### 4.4.4 Risk Preferring

There are some individuals who actually prefer risky activities to risk-free ones. These individuals are called risk lovers possessing a risk preferring attitude. A utility function for such an agent is shown in Fig. 4.5. Note that the utility function in Fig. 4.5 becomes steeper as the agent's payoff increases. Hence, a risk-preferring agent has increasing marginal utility for additional units of payoff represented by convex utility function. This simply means that the risk preferring individual is quite willing to take on additional risk for the possibility of a higher valued payoff. To understand such behaviour better, consider an example of an individual who faces a choice between a risky activity A, with a 50% $\left(=\frac{1}{2}\right)$ chance of obtaining a payoff of Rs. 5 and a 50% $\left(=\frac{1}{2}\right)$ chance of obtaining Rs. 10, or go for a risk-free activity B with a certain payoff equal to the expected payoff of the risky activity A, i.e., Rs. 7.5 $\left(= \frac{1}{2} \times 5 + \frac{1}{2} \times 10\right)$.

Now assume this individual has utility function given by U (X) = $X^2$, where X is the payoff received. Here again we proceed in the similar way like we did in case of risk-neutral and a risk-averse individual.

U(5) = 25, the height at point *a*; U(10) = 100, height at the point *b*.

Expected Utility, i.e. E(U) of the risky activity A = $[\frac{1}{2} \times U(5) + \frac{1}{2} \times U(10)] = [\frac{1}{2} \times 25 + \frac{1}{2} \times 100] = 62.50$, height at point *c*, representing mean value of the line joining points a and b.

A risk-free activity B offering a certain payoff equal to the EV of the Risky activity A, *i.e.* Rs. 7.5, will also have an EV of Rs. 7.5 (as probability of a certain payoff = 1).

Utility attained from activity B, i.e. [U(EV)] = U(7.5)

= $(7.5)^2$ = 56.25, which is equal to the height at point d.

Here, you can easily notice from the figure, that height ce > de, that is, we get E(U) > U(EV). In other words, expected utility attained by this individual from risky activity A is greater than the utility he receives from risk-free activity B giving certain outcome (that is, 62.50 > 56.25). Such a behaviour exhibits that the individual is risk preferring, with expected utility associated with a risky choice being more than the utility attained from a certain outcome (= the expected outcome of the risky choice) of a risk-free choice.

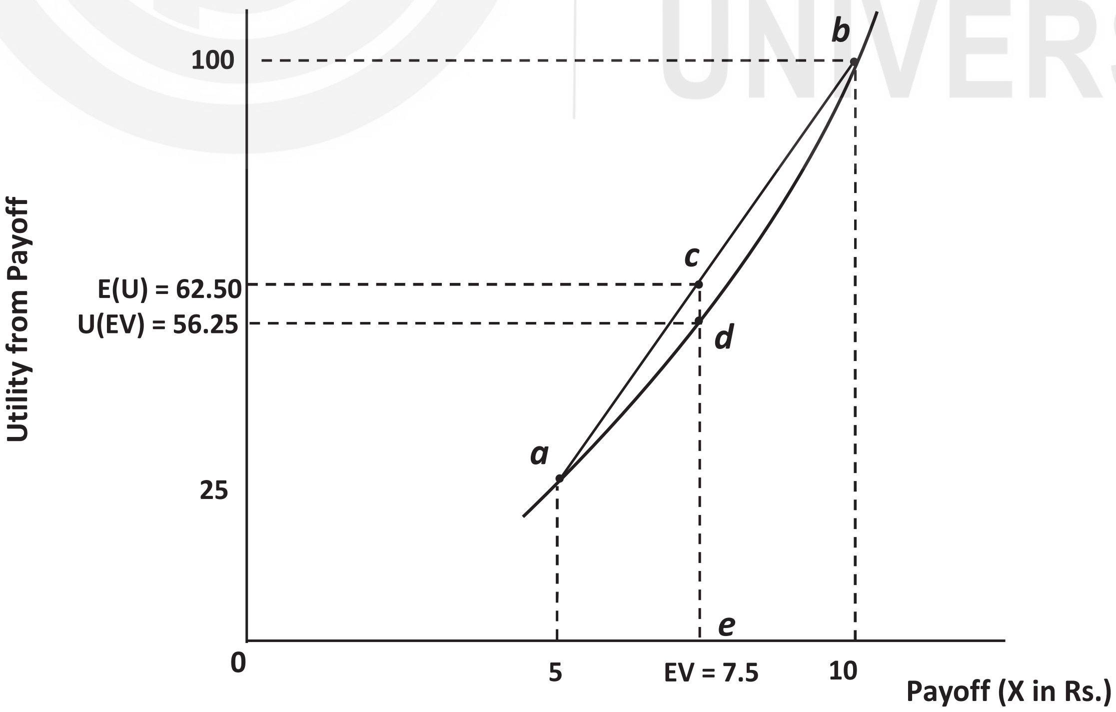


Fig. 4.5: Risk Preferring

## 4.5 RISK AVERSION AND INSURANCE

In addition to characterising an agent's attitude toward risk, expected utility theory can be of use to us in analysing more applied questions about insurance and about risk-taking in general. To understand the value of expected utility theory in such areas, let us consider Fig. 4.6.

Fig. 4.6 is identical to Fig. 4.4. It depicts the utility function of an agent who is averse to risk. Let us assume that this agent owns a house that has a current value of Rs. 100 and that she is aware of the possibility that the house may burn down, in which case the land it is on, will be worth Rs. 36 only. Let us also assume that from previous history, we know that there is a 20% chance that the agent's house will burn down. Therefore, we can say that during the next period, the agent is actually facing a gamble in which she will have a house worth Rs. 100 with a probability of 80% (=$\frac{4}{5}$) , or she will have land worth Rs. 36 with probability 20% (= $\frac{1}{5}$).

Expected value of the gamble $= \frac{4}{5} \times 100 + \frac{1}{5} \times 36$

= Rs. 87.2

Assuming individual's utility function to be U (X) = $\sqrt{X}$, where X is the rupee value.

U(36) = $\sqrt{36}$ = 6, the height at point *a*; U(100) = $\sqrt{100}$ = 10, height at the point b. Therefore, the expected utility E(U) is

$$E(U) = [\tfrac{1}{5} \times U(36) + \tfrac{4}{5} \times U(100)] \Rightarrow [\tfrac{1}{5} \times 6 + \tfrac{4}{5} \times 10] = 9.2,$$

height at point c, representing mean value of the line joining points a and b.

The utility from the expected value (or income) of the gamble U(EV) is:

U(EV) = U(87.2) = $\sqrt{87.2}$ = 9.338, height at point d on the utility function.

If the agent does nothing, her current state (ownership of the house) is worth the height cg (= 9.2) to her in terms of utility. Could this risk averse agent be assured of some certain amount (let it be denoted by Rs. C), which would give her a level of utility equivalent to the expected utility level from the risky gamble, i.e. 9.2? Point e on the utility function curve represents such a possibility.

The corresponding Rupee value on the *x*-axis associated with point e will be given by

U(C) = 9.2

$\sqrt{C}$ = 9.2 (squaring both sides to get the value of C)

C = 84.64

Note that cg and ef are the same height, where ef represents utility to the agent (= 9.2) with a sure payoff of Rs. 84.64. This amount of Rs. 84.64 is known as the *Certainty Equivalent* of the Gamble.

**Certainty Equivalent (CE)**

Certainty equivalent (CE) of a risky activity is the amount of money for which an individual is indifferent between the gamble and the certain amount. In other words, it is the amount of money received with certainty giving a utility level to the individual equivalent to the level attained by risky gamble. It is also called the *selling price*, from the fact that it serves as the definite price at which the activity could be sold, or at which individual will be indifferent between facing and selling a gamble.

Our agent can obtain Rs. 84.64 for sure if someone is willing to sell her insurance on the house for a yearly premium (price) of Rs. 2.56 (= Rs. 87.2 – 84.64). This 2.56 amount is nothing but what is called the *Risk Premium*. It is simply the amount which the agent is willing to forego in order to be indifferent in her choice between a risky gamble and the one with a certain return. The notion of a risk premium is directly applicable to insurance policies. An individual who purchases an insurance policy willingly pays a sum of money, known as an insurance premium (or Risk premium), in order to guarantee a certain level of monetary value generally associated with some type of risky activity.

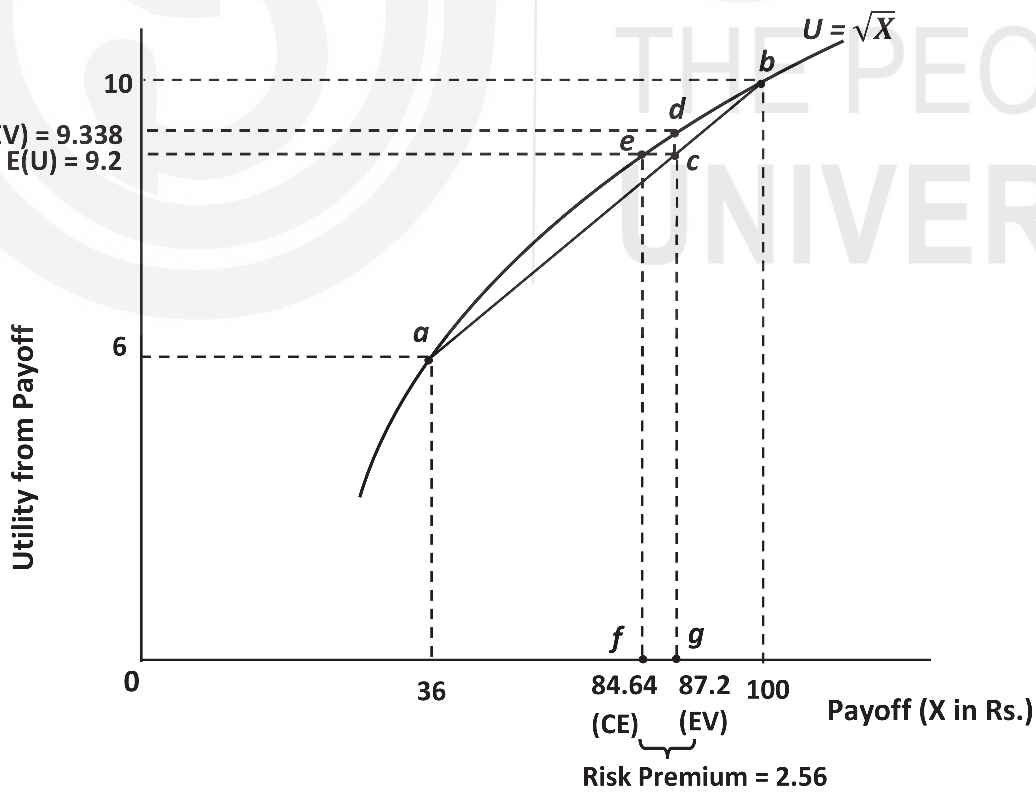


Fig. 4.6: Risk Aversion and Insurance

**Check Your Progress 2**

1) Define the concepts of Expected Value and Expected Utility. Explain their application in determining the attitude of an individual towards risk.

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2) A risk-averse individual is offered a choice between a gamble that pays Rs. 1000 with a probability of 25% and Rs. 100 with a probability of 75%, or a payment of Rs. 325. Which would he choose?

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3) How does insurance help reducing risk?

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## 4.6 INTERTEMPORAL DECISION-MAKING

The word "Intertemporal" simply means "across time". So far we have assumed that consumer exhausts all his income in a given time period and accordingly makes consumption decisions. We did not consider that income in one time period can be transferred to another time period, that is, our analysis so far has been static. In this section, we will attempt to look into decision-making process adopted by an individual across time periods. For instance, his decision about how to allocate his income through time: whether to borrow for current consumption or save for retirement; whether to build up a pension fund or not; whether to save for a holiday or spend all right away, etc.

### 4.6.1 Intertemporal Budget Constraint

Under intertemporal decision-making, individual decides about his savings and borrowings. To begin with this, let us first describe how the budget constraint changes when there are intertemporal decisions.

Up till now, the budget constraint we have been considering is given by,

$$p_1x_1 + p_2x_2 = M$$

Where, $p_1$ is the price of good $x_1$, $p_2$ is the price of good $x_2$ and total income of the consumer in the given time period is M. Along with this, we have been assuming that consumer exhausts all his income, so that total expenditure is equal to total income. Now with intertemporal choices, there is an intertemporal budget constraint.

Consider an individual who is assumed to live for two periods, 1 and 2, earning $Y_1$ and $Y_2$ amounts of income, respectively. Let us denote consumption of this individual in period 1 and 2 by $C_1$ and $C_2$, respectively. Along with consumption, individual also has an option to either save (then $C_1 < Y_1$) or borrow (then $C_1 > Y_1$) in period 1, with savings being given by $S_1 = Y_1 - C_1$ (and borrowing is negative saving). The market rate of interest (r) is assumed to be given and constant, which an individual could earn or pay on his savings or borrowings, respectively. If consumer saves $S_1$ amount in period 1, then in period 2 he would earn $(1 + r) S_1$ in income.

Individual under the above conditions is thus faced with the problem of choosing an optimal consumption stream ($C_1$ and $C_2$) through time, given the stream of income ($Y_1$ and $Y_2$) and market rate of interest. There are two possibilities:

If individual decide not to consume the entire $Y_1$ in period 1, i.e., $Y_1 > C_1$, then $S_1 > 0$ (individual saves $Y_1 - C_1$ in period 1). Then the budget constraint of this individual for period 1 will be given by,

$$C_1 + S_1 = Y_1$$

Whereas, if he decides to consume more than what he has in period 1, i.e., $Y_1 < C_1$, then $S_1 < 0$ (individual borrows $C_1 - Y_1$ in period 1). This individual will have to then pay back $(1 + r) S_1$ amount to the lender in period 2. The budget constraint of this individual for period 2 will be given by,

$$C_2 = Y_2 + (1 + r) S_1$$

These two budget constraints can be combined into one by solving for $S_1$. The intertemporal budget constraint then will be given by,

$$Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r}$$

This means that, if we assume that there are two time periods, then total lifetime income (i.e., $Y_1 + \frac{Y_2}{1+r}$) is equal to total lifetime consumption (i.e., $C_1 + \frac{C_2}{1+r}$). Which is highlighting the fact that an in lifetime individual cannot consume more than his income. Income in period 2 is discounted by the factor (1 + r) to get the present value of future income, this is what is done with consumption in period 2. Thus, the intertemporal budget constraint says that the present discounted value of consumption expenditures must equal the present discounted value of income.

Following is the diagrammatic representation of this budget constraint. (Fig. 4.7).

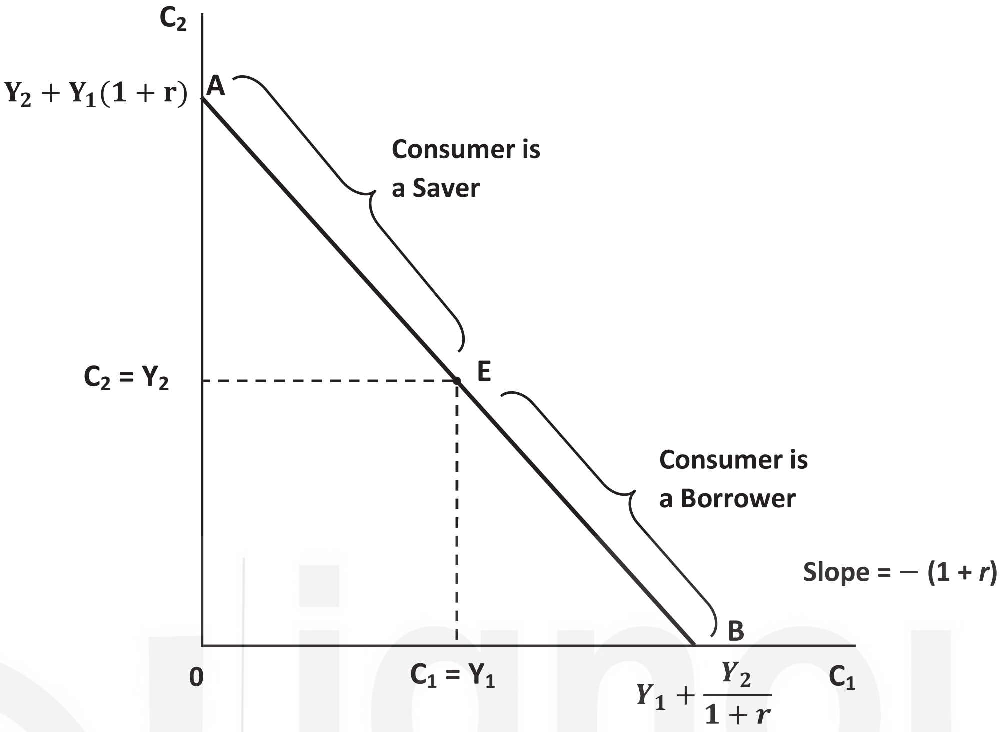


**Fig. 4.7: Intertemporal Budget Constraint**

Graphed in ($C_1$, $C_2$) plane, the intertemporal budget constraint is a straight line (here AB) with slope − (1 + r). In the above diagram, E is the point of endowment, which means, consumption in each time period is equal to respective income. If the consumer chooses a point to the right of the point E (i.e. in the segment EB), then he is a borrower, as he borrows to consume in period 1, i.e., $C_1 > Y_1$. If he consumes his lifetime income in period 1, then maximum he can consume in period 1 is $Y_1 + \frac{Y_2}{1+r}$ , the intercept on horizontal axis. Similarly, if he consumes in any point to the left of the endowment point (i.e. in the segment AE), then he is a saver in period 1, i.e., $C_1 < Y_1$. If he saves all his income in period 1, then maximum in period 2 is $Y_2 + Y_1 (1 + r)$, the intercept on vertical axis.

Thus intertemporal budget constraint shows all the possible combinations of consumption in two periods ($C_1$ and $C_2$) available to the consumer, given $Y_1$, $Y_2$ and r. Given this constraint, the consumer attempts to reach his optimal by placing his preferences over these allocations. With increase in the rate of interest, consumption today becomes more expensive than consumption in the future, hence, the budget constraint becomes steeper around the endowment and vice versa.

### 4.6.2 Preferences over Two Time Periods: Indifference Curves

How does a consumer place his preference over consuming today or tomorrow? Each individual has different preferences. Some might want to consume all today while some might want to save all today. There might be some who would want to save some today to enjoy higher consumption in future, while there are others who enjoy greater consumption today through borrowing and thus are left with lesser income to consume in future.

Such differences show that consumer places his preferences over consuming in two time periods like placing preferences between two goods. Here consumption in both the time periods (*i.e.* $C_1$ and $C_2$) will be treated as "normal goods", as consumer decides to consume in both the time periods. This assumption implies that indifference curves showing different combinations of $C_1$ and $C_2$, keeping consumer utility constant, to be downward (negative) sloping and convex-shaped ($IC_1$, $IC_2$ and $IC_3$ in the Fig. 4.8). Downward or a negative slope of IC implies that an increase in consumption in one period must be accompanied by decrease in consumption in another period, so as to keep the satisfaction level constant. The idea of convex indifference curves have been studied in detail earlier. Convex preferences are a reflection of a decreasing marginal rate of substitution, which simply means that, as consumption in any one period increases more and more, the individual will prefer to sacrifice (substitute) lesser and lesser amounts of consumption in the other period. Fig. 4.8, shows such preferences.

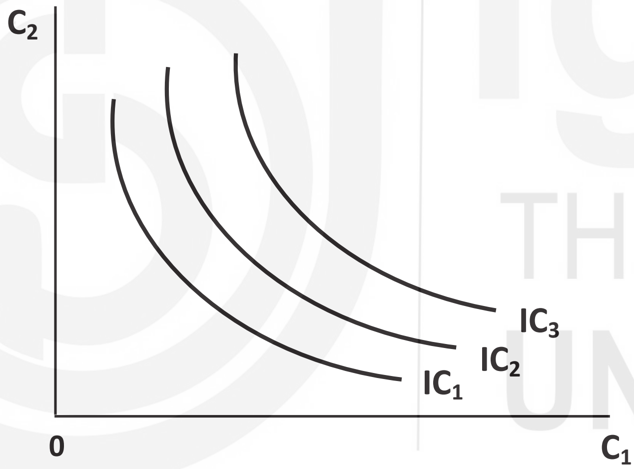


**Fig. 4.8: Indifference Curves**

How does consumer choose between two time periods, in other words, how consumer allocates his/her consumption over time? Through intertemporal budget constraint, all the allocation possible to him/her at the given market interest rate "r" are derived, and through indifference curves, his/her preferences over two time periods are described. Consumer optimal is achieved where the intertemporal budget constraint is tangent to the convex indifference curve (refer Fig. 4.9). At the point of tangency, diminishing slope of indifference curve is equal to the constant slope of intertemporal budget constraint. Thus optimal consumption between two time periods ($C_1$*, $C_2$*) occurs where, $-\frac{\partial U/\partial C_1}{\partial U/\partial C_2} = -(1+r)$

Marginal Rate of Substitution (Slope of Indifference curve) is given by $-\frac{\partial U/\partial C_1}{\partial U/\partial C_2}$, and slope of budget constraint is – $(1+r)$.

The point of tangency shows consumer choice. The tangency condition is necessary and sufficient condition to achieve equilibrium, as the preferences are convex.

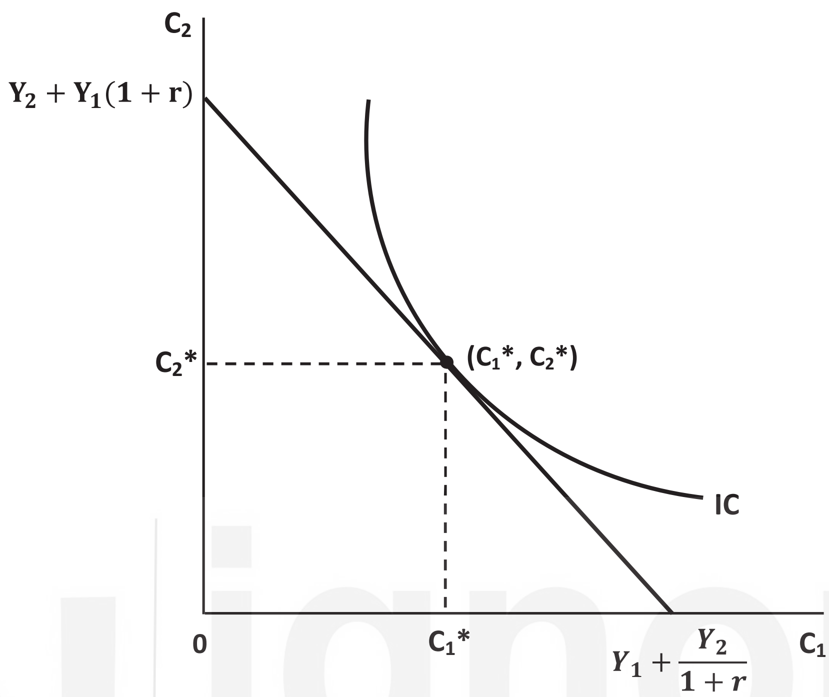


Fig. 4.9: Optimal Consumption Bundle in Two Periods

### 4.6.3 Case of a Borrower and a Lender

As we have discussed above, a consumer can choose to save in period 1 by consuming less than his income for that period, or he may consume more than the amount earned by borrowing in that period. Now to see whether the consumer chooses to become saver or borrower in time-period 1, consider Fig. 4.10 and 4.11 below:

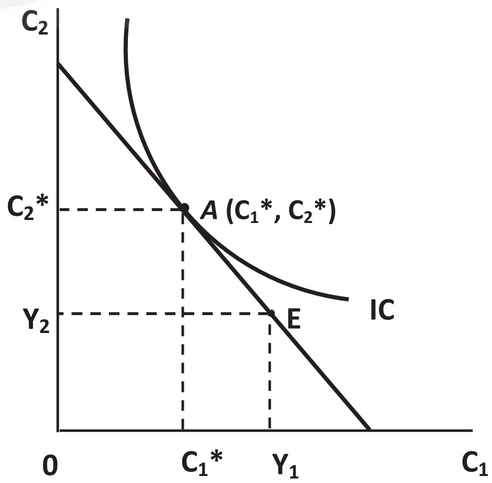


Fig. 4.10: Case of a Saver

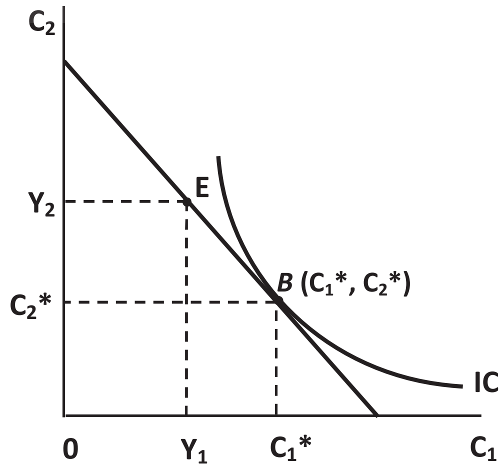


Fig. 4.11: Case of a Borrower

Fig. 4.10, shows the case of saver. Endowment point in given by point E, where $C_1 = Y_1$ and $C_2 = Y_2$. We can see here that the choice made ($C_1$*, $C_2$*) is to the left of the endowment (E) given by the tangency of the indifference curve and the intertemporal budget constraint. That is, the consumer chooses to consume at point "A" where, $C_1^* < Y_1$. Hence, this consumer saves in period 1 and will enjoy greater consumption in period 2.

Fig. 4.11, shows the case when consumer chooses to become borrower in period 1. Optimal allocation is given by point B (the tangency of the constraint and the indifference curve), where $C_1^* > Y_1$. Hence, the consumer will end up borrowing in period 1 and will have lesser consumption in period 2.

**Changes in Interest Rates**

Finally, let us look at what happens when interest rate "r" changes. First we consider the case when interest rate increases and how this affects consumer choice? This will be studied under two cases, (a) when individual initially is a saver, and (b) when he initially is a borrower.

Consider him a saver first (i.e., $C_1^0 < Y_1$), with an initial consumption bundle given by point A ($C_1^0$, $C_2^0$) in Fig. 4.12. When interest rate (r) rises, the intertemporal budget line (RU initially) pivots around the endowment point (E) and becomes steeper (TS ultimately). Pivoting of the new intertemporal budget line around the endowment point indicates that the individual can always consume the endowment in each period regardless of what "r" is. The horizontal-axis intercept shifts in to indicate the increased opportunity cost of present consumption resulting from increased interest rate. In contrast, the vertical axis intercept must shift up. Now to reach at the new optimal consumption in two periods bundle, we will be considering both, the substitution and the income effect of an increase in interest rate. Substitution effect will cause a fall in $C_1$ and an increase in $C_2$ resulting from rise in relative price of present consumption (1 + r). $C_2$ will definitely rise, but nothing can be said about $C_1$. This is due to income effect. An increase in r, with individual being a saver initially will result in a bigger return on his savings and hence more income in the next period. Since consumer now has greater lifetime income, and consumption in both the time period are considered to be normal goods, both $C_1$ and $C_2$ will increase. For $C_2$, both substitution effect and income effect works in same direction. But for $C_1$, they move in opposite direction. Substitution effect decreases $C_1$ while income effect increases it. Finally whether $C_1$ increases or decreases will depend on which effect is more dominating. Hence the resultant change in $C_1$ is ambiguous. Thus, if consumer is a saver and interest rate goes up, he will continue to be a saver (increased $C_2$ will ensure consumer stays on the left of the endowment point). This is illustrated in Fig. 4.12, where we assumed $C_1$ decreased with substitution effect dominating income effect, for us to be reaching at new optimal consumption bundle ($C_1$*, $C_2$*).

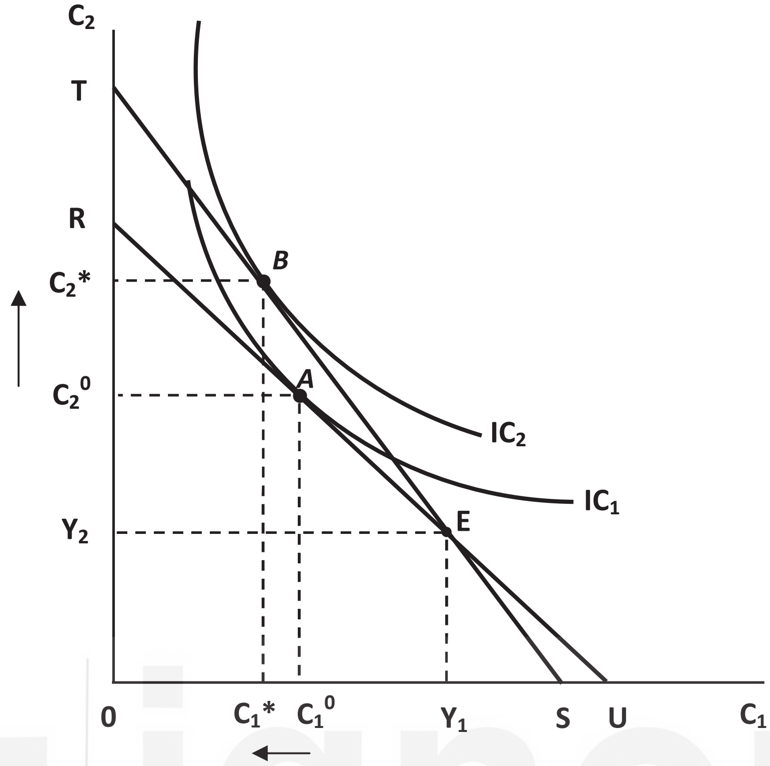


**Fig. 4.12: Consumer initially a Saver and Interest Rate rises**

Let us consider the case when interest rate rises and individual was a borrower in time period 1 (i.e. $C_1^0 > Y_1$). It will become expensive to consume in first period as relative price of $C_1$ (i.e., 1 + r) will rise, thus substitution effect will make $C_1$ to fall and $C_2$ to rise like before. An increase in r, with individual being a borrower initially will result in him paying back more on its borrowing, reducing his income in period 2 and hence his lifetime income. Reduced income will induce him to reduce both $C_1$ and $C_2$. Hence, for a borrower, the income and substitution effects go in the same direction, leading the individual to definitely reduce $C_1$. But here the result is ambiguous for $C_2$, as both, income and substitution effects go in opposite direction.

## Check Your Progress 3

1) A consumer, who is initially a lender, remains a lender even after a decline in interest rates. Is this consumer better off or worse off after the change in interest rates? If the consumer becomes a borrower after the change, is he better off or worse off?

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2) What is the present value of Rs. 100 one year from now if the interest rate is 10%? What is the present value if the interest rate is 5%?

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3) As the interest rate rises, does the inter-temporal budget constraint become steeper or flatter? Give reason.

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## 4.7 LET US SUM UP

In the earlier units we analysed the rational consumer's choice with the assumption of perfect information. However, in a real world situation, perfect information is far from reality. Such lack of perfect information in turn results in uncertainties or the situations involving risk. We began with some real life examples of consumers' choice under uncertainty and then introduced the concept of risk in a framework of probability distribution. On the basis of this the method of finding the expected value of the outcome associated with a risky/uncertain situation was explained. We proceeded with our model on consumer choice under uncertainties by bringing into picture the von Neumann-Morgenstern expected utility function, giving the average utility, or the expected utility, of the pattern of consumption in uncertain states. In addition to this, we elaborated the procedure involved in classification of individuals on the basis of their attitudes towards risk by comparing Expected Utility of the risky activity [E(U)] with Utility from expected value of a risk-free activity [U(EV)]. On the basis of such comparison, we came across three types of individuals with differing attitude towards risk— Risk Neutral, Risk Averse, and Risk Preferring. The unit further analysed the importance of Insurance for a Risk Averse individual. With the help of the concept of Certainty Equivalent, we discussed the concept of Risk Premium— the amount that an individual is willing to pay in order to guarantee a certain level of monetary value generally associated with some type of risky activity.

In further sections, we engaged ourselves to get an insight into the intertemporal decision-making by an individual. We saw how an individual choose an optimal consumption stream through time. This was achieved with the help of intertemporal budget constraint, which simply equated present value of the consumption stream with the present value of the income stream. We combined the budget constraint confining to two time periods, with the Indifference curves giving preferences of an individual over consumption in the two periods— to get to the optimal intertemporal choice, given the income stream and the market rate of interest. Lastly, we discussed the case a lender and a borrower by comparing an individual's consumption with his income in each period. We further explored how an interest rate increase affects a lender's or a borrower's intertemporal consumption.

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## 4.9 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

### Check Your Progress 1

1) Functions (a) and (c) have the expected utility property while (b)does not.

2) See Section 4.2 and answer.

3) Rs. 8,000

4) See Sub-section 4.3.1 and answer.

### Check Your Progress 2

1) **Hint:** Utility of the expected value [U(EV)] of a gamble is compared with the expected utility [E(U)] of the same gamble to infer about the individual's attitude toward risk. If U(EV) > E(U), then individual is risk averse; if U(EV) < E(U), then he is risk preferring, and if U(EV) = E(U), individual is risk neutral.

2) Here both, the gamble and the payment offer equal expected payoff of Rs. 325. Since the individual is risk-averse, he will prefer the risk-free expected value of the gamble that is Rs. 325 which he receives as payment, to the gamble itself.

3) See Section 4.5 and answer.

## Check Your Progress 3

1) See Sub-section 4.6.3 and answer.

   **Hint:** Fall in the interest rate will result in fall in consumption in second period as both substitution and income effects will work in same direction to reduce it, whereas for consumption in period one, they work in opposite direction. When consumer decides to remain a lender, then he will be worse off by settling down with a lower utility level. Whereas, there is scope of attaining a higher utility level by switching his behaviour to be that of a borrower after the interest rate fall.

2) Present value with 10% interest rate = $\frac{100}{1+0.1} = \frac{100}{1.1} = 91$ (Approx.) and if interest rate is 5% then it is 95 (Approx.).

3) Steeper, as consumption in period 1 becomes relatively expensive.

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