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School of Social Sciences
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BECC-109
INTERMEDIATE MACROECONOMICS - II

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INTERMEDIATE MACROECONOMICS - II

School of Social Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

EXPERT COMMITTEE

COURSE PREPARATION TEAM

Block/ Unit Title		Unit Writer
Block 1	**Economic Growth**	
Unit 1	Harrod-Domar Model	Dr. Archi Bhatia, Associate Professor, Central University of Himachal Pradesh, Dharamshala.
Unit 2	Solow Model	
Unit 3	Endogenous Growth Model	
Unit 4	Business Cycle	
Block 2	**Microeconomic Foundations**	
Unit 5	Inter-temporal Choice - I	Ms. Baishakhi Mondal, Assistant Professor, Indraprastha College for Women, University of Delhi
Unit 6	Inter-temporal Choice - II	
Unit 7	Investment Function	
Unit 8	Demand for Money: Post-Keynesian View	
Block 3	**Fiscal and Monetary Policy**	
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Unit 10	Monetary Policy	
Block 4	**Schools of Macroeconomic Thought**	
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Unit 12	Evolution of Macroeconomic Thought - II	

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CONTENTS

COURSE INTRODUCTION

This course is a sequel to BECC 106: Intermediate Macroeconomics I. In this course, the students are introduced to the long run dynamic issues such as growth and technical progress. It also provides the micro-foundations to various economic aggregates used in the previous course. There are four blocks in this course.

Block 1, entitled **Economic Growth**, comprises four Units. Unit 1 deals with the Harrod-Domar Model and the knife-edge problem. Unit 2 presents the Solow Model, with its neoclassical features and steady state growth. The Unit also discusses golden rule given by Phelps. Unit 3, entitled Endogenous Growth Model, discusses some of the simple growth models such as the AK model and the Romer model. Unit 4 presents an outline of the characteristics of Business Cycle.

Block 2 entitled **Microeconomic Foundations** comprises four Units. This block deals with the inter-temporal aspects of consumption and investment behaviour. Unit 5 and Unit 6 present the importance of Inter-temporal Choice in consumption. These two Units bring out the inconsistencies between short run and long run consumption functions, and the theories that attempt to reconcile such discrepancies. Unit 7 highlights the determinants of Investment Function. Unit 8 introduces you to the post-Keynesian views on Demand for Money.

Block 3, entitled **Fiscal and Monetary Policy**, consists of two Units. Unit 9, titled Fiscal Policy discusses issues such as policy lags, budget constraint, debt sustainability, and Ricardian equivalence proposition. Unit 9 on Monetary Policy highlights issues such as rules versus discretion, time consistency, Taylor's rule, and inflation targeting.

In Block 4 we discuss issues pertaining to **Schools of Macroeconomic Thought.** There are two Units in this Block. Both the Units put together provide and understanding of the theoretical developments in macroeconomics over time. This block covers important schools of thought such as Classical, Keynesian, Neoclassical, Monetarist, New Classical and New Keynesian.

UNIT 1 THE HARROD-DOMAR MODEL*

Structure

1.0 OBJECTIVES

After reading this unit, you will be able to

- state the necessity of sustained economic growth;
- outline the implications of the assumptions made in the Harrod Domar model;
- determine how steady state growth can be achieved in an economy with fixed saving rate and capital-output ratio;
- determine the conditions under which a steady growth rate can be maintained;
- distinguish between warranted growth rate and actual growth rate;
- discuss the instability problem of the Harrod-Domar Model; and
- identify the limitations of the Harrod-Domar Model.

1.1 INTRODUCTION

Of all the issues facing development economists, none is as compelling as the question of economic growth. A country's ability to provide improving standards of living for its people depends crucially on its long-run rate of economic growth. Over a long period of time, even an apparently small difference in the rate of economic growth can translate into a large difference in the income of the average person. Ever since the end of Second World War, interest in the problems of economic growth has led economists to formulate growth models of different types.

* Dr. Archi Bhatia, Associate Professor, Department of Economics and Public Policy, Central University of Himachal Pradesh, Dharamshala.

A feature common to them all is that they are based on the Keynesian saving-investment analysis. The Harrod-Domar Model is the first and the simplest model of economic growth.

You may recall that the Keynesian analysis was for the short run. If we extend it to the long run, we find that capital stock of a country increases as investment is more than the replacement investment or the depreciation level. Increase in capital stock leads to increase in production capacity of an economy. As production capacity increases, there is economic growth of the country. Thus, the Harrod-Domar model is a direct outcome of the projection of the short-run Keynesian analysis into the long-run.

1.2 FEATURES OF MODERN ECONOMIC GROWTH

Throughout most of human history, appreciable growth in per capita gross domestic product (GDP) was the exception rather than the rule. Let us consider the growth rates of the world's leading economies over the past four centuries. During the period 1580-1820, the Netherlands was a leading industrial country; it experienced an average annual growth in real GDP per worker hour of roughly 0.2 per cent. Average annual growth rate of the United States of America during the period 1890-1989 was a relatively dramatic 2.2 per cent per year. Although an annual growth rate of 2 per cent in per capita GDP does not appear very impressive, a moment's reflection (and calculation) reveals its enormous potential if such growth rate is sustained. Simple calculations show that at the 2 per cent rate, a country's per capita GDP doubles in 35 years – a length of time considerably shorter than the life span of an individual.

Robert Lucas, in his Marshall Lectures at the University of Cambridge in 1984-85 (prior to the acceleration in economic growth rate of India, when per capita income was growing at less than 1.5 per cent per annum) stated:

"Rates of growth of real per-capita income are….diverse, even over sustained periods…..Indian incomes will double every 50 years; Korean every 10 years. An Indian will, on average, be twice as well off as his grandfather, a Korean 32 times…..".

A sustained economic growth in the last century was not experienced the world over. In the nineteenth and twentieth centuries, only a handful of countries, mostly in Western Europe and North America could manage to "take off into sustained growth", to use a well-known term coined by the economic historian W.W. Rostow. Throughout most of what is commonly known as the Third World, the growth experience only began well into the twentieth century; for many of them, probably not until the post-World War II era, when colonialism ended.

The now-developed countries (such as the US, Canada, Australia, etc.) grew in an environment uninhabited by countries of far greater economic strength. Today, the story is completely different.

The developing countries not only need to grow, they must grow at rates that far exceed historical experience. The developed world already exists, and their access to economic resources is far higher than that of the developing countries. Exponential growth at rates of 2 per cent per annum may well have significant long-run effects, but they cannot match the parallel growth of human aspirations, and the increased perception of global inequalities.

1.3 UNDERLYING IDEAS

The Harrod-Domar Model was developed independently by Roy F. Harrod in 1939 and Evsey Domar in 1946. Although Harrod and Domar models differ in details, they are bracketed together because of their similarity of approach. Both these models emphasise the essential conditions of achieving and maintaining steady growth (i.e., occurring in a smooth, gradual and regular manner). Harrod and Domar assign a crucial role to capital accumulation in the process of growth. In fact, they emphasised the dual role of investment, viz., (i) it generates income through the multiplier effect, and (ii) it leads to capital accumulation and increase in productive capacity.

Economic growth is the result of abstention from current consumption. An economy produces a variety of commodities. The act of production generates income. The very same income is used to buy these commodities. Commodity production creates income, which creates demand for those very same commodities. We broadly classify commodities into two groups, viz., (i) *consumption goods*, which are produced for the purpose of satisfying human wants and preferences, and (ii) *capital goods*, which are produced for the purpose of producing other commodities. Mangoes, fountain pen, clothes, etc. are examples of consumption goods. A blast furnace, a conveyor belt, equipment, etc. come under the category of capital goods.

As you know from the circular flows of income and expenditure, the income generated from the production of all goods is spent on both consumer goods and capital goods. Typically, households buy consumer goods, whereas firms buy capital goods to expand their production or to replace worn-out machinery. All income is not spent on current consumption. By abstaining from consumption, households make available a pool of funds that firms use to buy capital goods. This is the act of investment. Note, however, that without the initial availability of saving it would not be possible to invest and there would be no expansion. This is the simple starting point of the theory of economic growth.

Implicit in this story is the idea of macroeconomic balance. If you think of a circuit diagram with income flowing out of firms as they produce and income flowing back into firms as they sell, you can visualize saving as a leakage from the system. The demand for consumption goods alone falls short of the income that created this demand. Investors fill this gap by stepping in with their demand for capital goods.

Macroeconomic balance is achieved when this investment demand is at a level that exactly counter-balances the saving leakage. This concept is summarized in Fig. 1.1, which depicts the circuit diagram.

Fig. 1.1: The Circuit Diagram: Production, Consumption, Saving and Investment

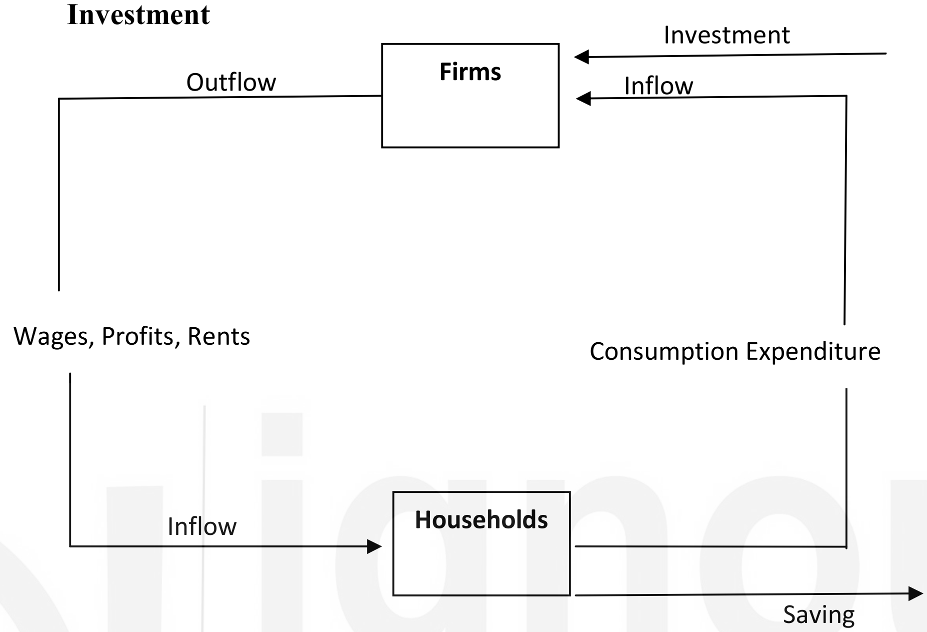

Fig. 1.1 shows income flowing out of the firms as they produce and income flowing back into the firms as they sell. Saving is the leakage from the system. Balance is reached when Investment exactly counterbalances the saving leakage.

Check Your Progress 1

1) Describe the importance of sustained economic growth.

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2) Explain the circuit diagram.

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1.4 ASSUMPTIONS OF THE MODEL

The main assumptions of the Harrod- Domar Model are as follows:

1) The economy is operating under full employment. It implies that there is no idle production capacity.
2) There is no government interference in the functioning of the economy.
3) The model is based on the assumption of "closed economy". In other words, government restrictions on trade and the complications caused by international trade are ruled out.
4) There are no lags in adjustment of variables, that is, economic variables such as saving, investment, income, expenditure, etc. adjust themselves completely within the same period.
5) The average propensity to save (APS) and marginal propensity to save (MPS) are equal to each other, i.e., APS = MPS. In symbols, $\boldsymbol{S/Y = \Delta S/\Delta Y = s}$**.** Households save a fixed proportion of their income every year.
6) The Capital Output ratio, θ, which is defined as the units of capital required to increase output by one unit is constant. Thus, $\boldsymbol{K/Y = \Delta K/\Delta Y} = \theta$.

   This amounts to assuming that the law of constant returns to scale operates in the economy because of the fixity of the capital-output ratio.
7) Income, investment and saving are all defined in the net sense, that is, they are considered over and above the depreciation. Thus, depreciation rates are not included in these variables.
8) The general price level is assumed to be constant, i.e., money income and real income are the same.
9) There are no changes in the interest rate.
10) There is fixed proportion of capital (K) and labour (L) in the production process.

These assumptions are meant to simplify the task of growth analysis, these could be relaxed later.

1.5 HARROD-DOMAR EQUATION

Economic growth is positive when investment exceeds the amount necessary to replace depreciated capital, thereby allowing the next period's cycle to recur on a larger scale. We adopt the following notations: *Y* denotes total Output, *C* denotes total consumption, and *S* denotes total saving. Remember that these variables are *aggregates* over the population. Thus the following equation shall be true as a matter of accounting:

$$Y_t = C_t + S_t, \quad \text{for all time periods } t \qquad \text{... (1.1)}$$

In other words, national income is divided between consumption and saving. The other side of the coin is that the value of produced output (also equal to Y) should be equal to the sum of consumption goods and capital goods. Thus,

$$Y_t = C_t + I_t \qquad ...(1.2)$$

where I denotes investment. If we equate equations (1.1) and (1.2), we obtain the famous macroeconomic balance equation, that is, saving equals investment. In notations,

$$S_t = I_t \qquad ... (1.3)$$

As we have mentioned earlier, investment augments the national capital stock, K. Thus,

$$K_t = K_{t-1} + I_t \qquad ... (1.4)$$

Equation (1.4) means that capital stock in period t equals the sum of capital stock in the previous period $(t-1)$ and the investment made in period t. We can write equation (1.4) as

$$I_t = K_t - K_{t-1} \qquad ... (1.4a)$$

By combining equations (1.3) and (1.4a) we obtain

$$S_t = K_t - K_{t-1} \qquad ... (1.5)$$

As defined earlier, saving rate $s_t = S_t/Y_t$. Therefore, $S_t = s_t Y_t$ or simply sY_t, if we assume saving ratio to be constant over time.

Similarly, $K_t = \theta Y_t$ and $K_{t-1} = \theta Y_{t-1}$ for all time periods t with θ as the capital output ratio.

Using these in (1.5), we get:

$$sY_t = \theta Y_t - \theta Y_{t-1} = \theta(Y_t - Y_{t-1}) \qquad ... (1.6)$$

By re-arranging terms in equation (1.6) we obtain

$$\frac{s}{\theta} = \frac{Y_t - Y_{t-1}}{Y_t} \qquad ... (1.7)$$

Or, $g_w = \frac{s}{\theta} = \frac{Y_t - Y_{t-1}}{Y_t}$ …(1.8)

where g_w is the required or warranted growth rate in National Income.

Equation (1.8) is the famous Harrod-Domar equation. According to this equation, for maintaining equilibrium in the economy, the ratio of 's' (saving-income ratio) to 'θ' (capital output ratio) should be equal to the growth rate ($g_w = \frac{Y_t - Y_{t-1}}{Y_t}$) in income or output of the economy.

The 'warranted growth rate' (g_w) refers to that growth rate of the economy when it is operating at full capacity. It can be interpreted as the growth rate required for full utilization of a growing stock of capital, so that entrepreneurs would be satisfied with the amount of investment actually made. The Harrod-Domar equation given at (1.8) links the growth rate of the economy to two fundamental variables: the ability of the economy to save, and the capital-output ratio. By pushing up the rate of saving, it would be possible to accelerate the rate of growth. Likewise, by increasing the rate at which capital produces output (a lower value of θ), growth can be enhanced. You may note that central planning in countries such as India and erstwhile Soviet Union, during the pre-liberalisation period, was deeply influenced by the Harrod-Domar equation. If an economy has to grow steadily, without any disturbances, then the two ratios (namely, saving rate and capital-output ratio) have to be constant over time, that is,

$$\frac{S_t}{Y_t} = \frac{S_{t-1}}{Y_{t-1}} = \frac{S_{t+1}}{Y_{t+1}} = \ldots\ldots\ldots\ldots\ldots\ldots.. = s, \text{ and}$$

$$\frac{K_t}{Y_t} = \frac{K_{t-1}}{Y_{t-1}} = \frac{K_{t+1}}{Y_{t+1}} = \ldots\ldots\ldots\ldots\ldots\ldots.. = \theta\ .$$

If such constancy holds, then the economy will grow steadily at the required rate of growth, which is equal to $\frac{s}{\theta}$.

A small amendment to the Harrod-Domar model allows us to incorporate the effects of population growth. It should be clear that as the equation (1.8) currently stands, it is a statement regarding the rate of growth of gross national product (GNP), not GNP per capita. To talk about per capita growth, we should net out the effects of population growth. This is easy enough to do. If population (P) growth rate is *n*, we have

$$n = \frac{P_t - P_{t-1}}{P_t} = 1 - \frac{P_{t-1}}{P_t} \qquad \ldots (1.9)$$

From equation (1.9) we find that

$$\frac{P_{t-1}}{P_t} = (1-n) \qquad \ldots (1.10)$$

Let $y_t = Y_t/P_t$ denote per capita income

From equation (1.6) we have $\theta Y_t = \theta Y_{t-1} + sY_t$. Let us divide both sides by P_t. Thus we have

$$\theta Y_t/P_t = \theta Y_{t-1}/P_t + sY_t/P_t \qquad \ldots (1.11)$$

$$\theta\, y_t = \frac{\theta Y_{t-1}}{P_t} * \frac{P_{t-1}}{P_{t-1}} + sy_t \qquad \ldots (1.12)$$

Recall that we denote y_t as per capita income while Y_t is total income of the country.

$$\theta\, y_t = \theta\, y_{t-1} * \frac{P_{t-1}}{P_t} + sy_t \qquad \ldots (1.13)$$

$$\theta\, y_t - \left(\theta\, y_{t-1} * \frac{P_{t-1}}{P_t}\right) = sy_t \qquad \ldots (1.14)$$

$$\theta\left[y_t - y_{t-1} * \frac{P_{t-1}}{P_t}\right] = sy_t \qquad \ldots (1.15)$$

Lest us divide both sides of equation (1.15) by y_t

$$1 - \frac{y_{t-1}}{y_t} * \frac{P_{t-1}}{P_t} = \frac{s}{\theta} \quad \text{... (1.16)}$$

$$\frac{y_{t-1}}{y_t} * \frac{P_{t-1}}{P_t} = 1 - \frac{s}{\theta} \quad \text{... (1.17)}$$

Let us define $\frac{y_t - y_{t-1}}{y_t} = g^*$ where g^* is the per capita growth rate. This can be rearranged to show that $1 - \frac{y_{t-1}}{y_t} = g^*$ or $\frac{y_{t-1}}{y_t} = (1 - g^*)$.

In equation (1.10) we have shown that $\frac{P_{t-1}}{P_t} = (1 - n)$ where n is the population growth rate. If we substitute these two expressions ($\frac{y_t - y_{t-1}}{y_t}$ and $\frac{P_{t-1}}{P_t}$) in equation (1.17), we obtain

$$(1 - g^*)(1 - n) = 1 - \frac{s}{\theta} \quad \text{... (1.18)}$$

We expand equation (1.18) and see that

$$1 - g^* - n + g^*n = 1 - \frac{s}{\theta}$$

$$g^* + n - g^*n = \frac{s}{\theta}$$

Since both g^* and n are small numbers, such as 0.05 or 0.02, their product is very small relative to the other terms and can be ignored as an approximation. This gives us the approximate equation

$$\boxed{\frac{s}{\theta} = g^* + n \quad \text{... (1.19)}}$$

This is an expression that combines some of the fundamental features underlying growth, viz., the ability to save and invest (captured by s), the ability to convert capital into output (which depends inversely on θ), and the rate of population growth (n).

Check Your Progress 2

1) State the equation for the warranted growth rate.

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2) Under what conditions can an economy grow steadily, without any disturbances?

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3) How does the Harrod-Domar equation change when the effect of population growth rate is incorporated?

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1.6 KNIFE-EDGE PROBLEM

Let us now discuss the issue: how to achieve steady growth? According to Harrod, the economy can achieve steady growth if and only if the expected growth rate (let us denote it by g_t^e) equals the warranted growth rate, g_w. What if the expectation is for some rate of growth other than g_w?

As you know from equation (1.8), $g_w = \frac{s}{\theta}$. We assume that capital output ratio is constant and is equal to Θ for every period.

$$\frac{K_t}{Y_t} = \frac{K_{t-1}}{Y_{t-1}} = \frac{K_{t+1}}{Y_{t+1}} = \dots\dots\dots\dots\dots\dots = \Theta$$

Or $K_t = \Theta Y_t$ or $K_{t+1} = \Theta Y_{t+1}$ … (1.20)

We know that

$$I_t = K_t - K_{t-1}$$

Substituting the value of K_t from equation (1.20) in the above gives us

$$I_t = \Theta(Y_t - Y_{t-1}) \quad \text{... (1.21)}$$

Since Y_t (output in current time period is known only after the period is over; before that it is just an estimate or expected value) is not known, the economic agents make an expectation about it, Y_t^e. Investment in period *t* will therefore depend upon Y_t^e (see Units 4 and 5 of BECC-106)

$$I_t = \Theta(Y_t^e - Y_{t-1}) \quad \text{... (1.22)}$$

Investment in period *t* depends on difference between expected demand in period *t* and actual demand in period $(t-1)$.

If $Y_t^e > Y_{t-1}$, $then\ I_t > 0\ and\ Y_t^e < Y_{t-1}$, $then\ I_t < 0$

From the Keynesian model we know that

$$Y_t = C_t + I_t \quad \text{... (1.23)}$$

In (1.23), aggregate demand is the sum of consumption and investment demand as there is no government sector and foreign trade by assumption.

A fixed fraction of income is consumed (since we assume *s* to be constant in the model).

$$C_t = cY_t \quad \text{... (1.24)}$$

Substituting C_t from equation (1.24) in equation (1.23), we get

$$Y_t = cY_t + I_t \quad \text{... (1.25)}$$

$$Y_t = \frac{1}{1-c} I_t \quad \text{... (1.26)}$$

$$Y_t = \frac{1}{s} I_t \quad \text{... (1.27)}$$

where $\frac{1}{s}$ is the investment multiplier with which income Y will go up when investment demand, *I,* increases by one unit.

If we divide both sides of equation (1.27) by $\mathrm{Y_t^e}$, we get

$$\frac{Y_t}{\mathrm{Y_t^e}} = \frac{1}{s}\frac{I_t}{\mathrm{Y_t^e}} \quad \text{... (1.28)}$$

Substituting for I_t from equation (1.22) in equation (1.28), we obtain

$$\frac{Y_t}{\mathrm{Y_t^e}} = \frac{\theta}{s}\frac{[\mathrm{Y_t^e} - Y_{t-1}]}{\mathrm{Y_t^e}} \quad \text{... (1.29)}$$

Since $\frac{\mathrm{Y_t^e} - Y_{t-1}}{\mathrm{Y_t^e}} = g_t^e,$ the expected growth rate, we have

$$\frac{Y_t}{\mathrm{Y_t^e}} = \frac{\theta}{s} g_t^e \quad \text{... (1.30)}$$

Since $\frac{s}{\theta}$ is the warranted growth rate, g_w, we have:

$$\frac{Y_t}{\mathrm{Y_t^e}} = \frac{g_t^e}{g_w} \quad \text{... (1.31)}$$

An implication of (1.31) is that the expected growth rate will be equal to the warranted growth rate, *if and only if* the expected output $\mathrm{Y_t^e}$ is equal to the actual output Y_t. It means that steady growth is possible only if the expected output $\mathrm{Y_t^e}$ is equal to the actual output Y_t. In symbols,

$$g_t^e = g_w = \frac{s}{\theta} \ \textit{if and only if} \ \mathrm{Y_t^e} = Y_t$$

Since expectations to be correct is a matter of pure chance, according to Harrod-Domar model, realisation of steady growth is uncertain.

Let us look again into the actual growth rate, g_t^a.

$$g_t^a = \frac{Y_t - Y_{t-1}}{Y_t} = 1 - \frac{Y_{t-1}}{Y_t} \quad \text{... (1.32)}$$

Our objective is to find a relationship between g_t^a and g_t^e, so that both the rates can be compared.

$$g_t^e = \frac{\mathrm{Y_t^e} - Y_{t-1}}{\mathrm{Y_t^e}} = 1 - \frac{Y_{t-1}}{\mathrm{Y_t^e}}$$

Or, $\frac{Y_{t-1}}{\mathrm{Y_t^e}} = (1 - g_t^e)$ ... (1.33)

By re-arranging terms in (1.33) we find that

$$Y_{t-1} = (1 - g_t^e)\mathrm{Y_t^e} \quad \text{... (1.34)}$$

Let us look back at equation (1.31).

We have $\frac{Y_t}{Y_t^e} = \frac{g_t^e}{g_w}$, which we can re-arrange as

$Y_t = \frac{g_t^e}{g_w} Y_t^e$ or, $Y_t = \frac{g_t^e}{s/\theta} Y_t^e$. This gives us

$$Y_t = \frac{\theta}{s} g_t^e Y_t^e \qquad \text{... (1.35)}$$

Now we have values of Y_{t-1} from equation (1.34) and value of Y_t from equation (1.35). Let us substitute these values in equation (1.32). We find that

$$g_t^a = 1 - \frac{(1-g_t^e) Y_t^e}{\frac{\theta}{s} g_t^e Y_t^e} \qquad \text{... (1.36)}$$

Thus,

$$g_t^a = 1 - \frac{(1-g_t^e)}{g_t^e} * \frac{s}{\theta} \qquad \text{... (1.37)}$$

When the actual growth rate is equal to expected growth rate (i.e., $g_t^e = g_t^a$), we have from equation (1.37)

$$g_t^a = 1 - \frac{(1-g_t^a)}{g_t^a} * \frac{s}{\theta}$$

Or, $(1 - g_t^a) = \frac{(1-g_t^a)}{g_t^a} * \frac{s}{\theta}$

Or, $(1 - g_t^a) g_t^a = (1 - g_t^a) * \frac{s}{\theta}$

Hence, $g_t^a = \frac{s}{\theta}$ ...(1.38)

It is clear that g_t^a (actual growth rate) equals $\frac{s}{\theta}$, the warranted growth rate (required for steady growth of the economy) *if and only if* the actual growth rate is equal to the expected growth rate, g_t^e. That is

$$g_t^a = g_t^e = g_w = \frac{s}{\theta}$$

Actual growth rate = Expected growth rate = warranted growth rate = $\frac{s}{\theta}$

We already know from previous discussion that expected growth rate will equal $\frac{s}{\theta}$ if and only if the expectations are correct, that is $Y_t^e = Y_t$.

According to the Harrod-Domar model, there will be instability in an economy if actual growth rate deviates from the expected growth rate. If $g_t^e > \frac{s}{\theta}$ actual growth rate will be higher than expected growth rate. On the other hand, if $g_t^e < \frac{s}{\theta}$ actual growth rate will be lower than expected growth rate.

This in fact is the beginning of the Harrod's instability problem. The steady state growth of the economy requires equality between $g_t^a = g_t^e = g_w = \frac{s}{\theta}$

In a free enterprise economy, these equilibrium conditions would be satisfied only rarely, if at all. Therefore Harrod analyzed the situations when these conditions are not satisfied. If investors anticipate more than the warranted rate of growth $\frac{s}{\theta}$, then actual growth rate will exceed even the high expected growth rate.

In other words, if g_t^e exceeds $g_w = \frac{s}{\theta}$, then the actual growth rate will turn out to be even greater than the already high expected growth rate. Investors would then get a wrong signal. Instead of feeling that they expected too much, and slowing down their investment, investors will feel that they expected too little since $g_t^a > g_t^e$. So in the next period, they will invest more. Thus the gap between the actual growth rate and the warranted growth rate will keep widening in each period (see Fig. 1.2). The economy will experience inflation. There will be insufficient goods in the pipeline or insufficient equipment. Such a situation will lead to secular inflation because actual income grows at a faster rate than the one allowed by the growth in the productive capacity of the economy. In other words, the following inequality holds $g_t^a > g_t^e > g_w = \frac{s}{\theta}$.

Fig. 1.2: Interaction between Actual, Expected and Warranted Growth Rates

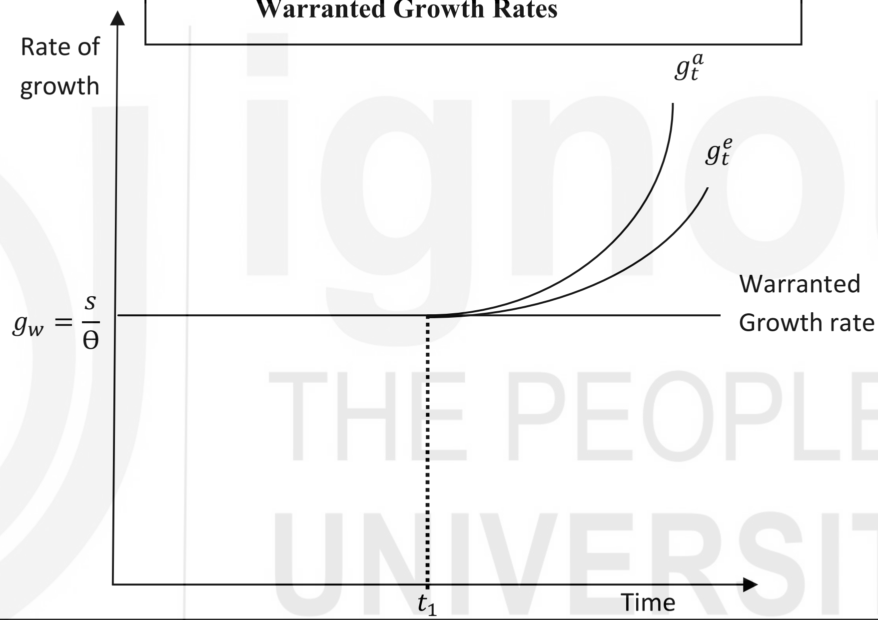

Fig. 1.2 shows that when g_t^e exceeds $g_w = \frac{s}{\theta}$ then actual growth rate will turn out to be even greater than the already high expected growth rate. Investors would then get a wrong signal. Such a situation will lead to secular inflation because actual income grows at a faster rate than allowed by the growth in the productive capacity of the economy. The following inequality holds $g_t^a > g_t^e > g_w = \frac{s}{\theta}$.

On the other hand, if investors anticipate a growth rate lower than the warranted growth (i.e., $g_t^e < g_w$) then actual growth rate will be even lower than the expected growth rate. In other words, $g_t^a < g_t^e < g_w$. The investors will now feel that they expected too much as the actual growth rate turns out to be lower than the expected growth rate. So in the next period, they will invest even lesser thereby further widening the gap between actual and warranted growth rates (see Fig. 1.3). Such a situation will lead to secular depression because actual income grows much slower than what is required for full utilization of the productive capacity of the economy.

This is the crux of Harrod's problem. A slight deviation of actual growth rate from warranted growth rate leads the economy drift farther away from the steady state growth path. It is hence called "knife-edge equilibrium". Fig. 1.2 and Fig. 1.3 show the interaction between actual, expected and warranted growth rates.

Fig. 1.3: Interaction between Actual, Expected and Warranted Growth Rates

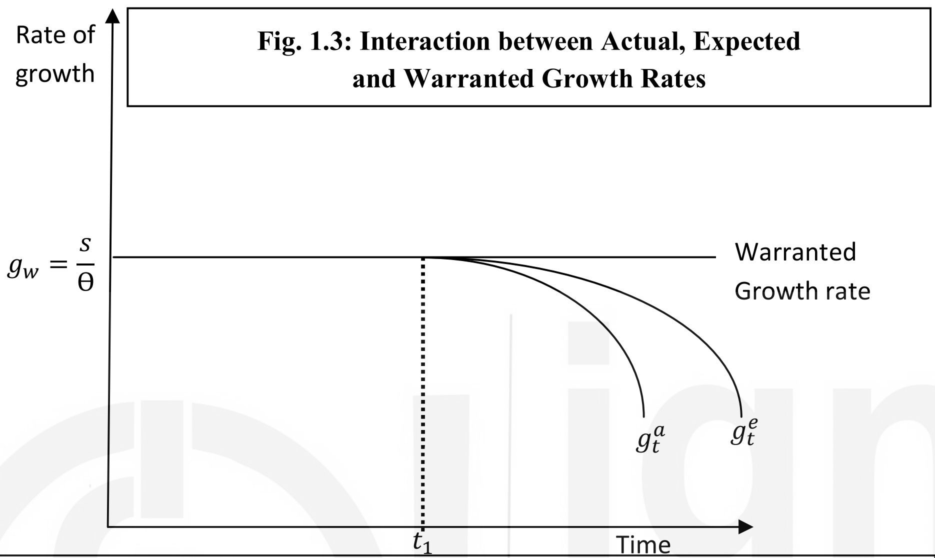

Fig. 1.3 shows that when $g_t^e < g_w$, then actual growth rate will be even lower than the expected growth rate. In other words, $g_t^a < g_t^e < g_w$. Such a situation will lead to secular depression because actual income grows slowly than what is required for full utilization of the productive capacity of the economy.

An Illustration:

Let us understand the Harrod's instability problem through an example. For an economy, let the saving rate be 20 per cent (s=0.2); the capital-output ratio $\Theta = \frac{\Delta K}{\Delta Y} = 2$. Hence the warranted growth rate will be

$$g_w = \frac{s}{\Theta} = 10\% = 0.1$$

Suppose, the previous period's output is given as $Y_{t-1} = 90$. The economy can attain steady state equilibrium when the actual output growth rate equals expected output growth rate which in turn equals the warranted growth rate of 10 per cent. Thus for steady state growth, the expected output should be equal to the actual output of 100 units, $Y_t^e = Y_t = 100$.

If the above holds then, $g_t^a = \frac{Y_t - Y_{t-1}}{Y_t} = \frac{100-9}{100} = 10\%$.

In this case $g_t^a = g_w = 10\%$.

Here investment will equal, $I = \Theta\Delta Y = 2 * 10 = 20$ units

Now this investment will create aggregate demand through the multiplier effect, which equals $Y = \frac{1}{s} I$

$Y = \frac{1}{0.2} * 20$

$Y_t = 100$

Thus if investors expect an output of 100 units, they will invest 20 units in trying to create capacity for an additional 10 units of demand, $(Y_t - Y_{t-1})$. This investment of 20 units will generate through the multiplier of 5 yielding an aggregate demand of 100 units. In this way, expectations are realized, that is, $g_t^e = g_t^a = g_w = \frac{s}{\Theta} = 10\%$.

If, however, the investors anticipate a little too much, i.e.,$Y_t^e = 101$. The additional units which are expected to be produced, $Y_t^e - Y_{t-1} = 101 - 90 = 11$ units.

The investors will then invest, $I = \Theta\Delta Y = 2 * 11 = 22$ units.

Now, these 22 units of investment will create an aggregate demand of

$Y = \frac{1}{0.2} * 22$

$Y_t = 110$

As can be seen from above, $Y_t > Y_t^e$, that is, 110 >101.

Let us now calculate the actual and expected growth rates.

$g_t^a = \frac{Y_t - Y_{t-1}}{Y_t} = \frac{110-90}{110} = 18.18\%$

$g_t^e = \frac{Y_t^e - Y_{t-1}}{Y_t^e} = \frac{101-90}{101} = 10.8\%$

As can be seen now,

$g_t^a > g_t^e > g_w$

$18.18\% > 10.8\% > 10\%$

Investors in the next period will feel that they invested too little and will further increase their investments which in turn will further widen the gap.

Let us now look into the opposite situation. Suppose, the investors expect too little, $Y_t^e = 99$. The additional units which are expected to be produced, $Y_t^e - Y_{t-1} = 9$ units.

The investors will then invest, $I = \Theta\Delta Y = 2 \times 9 = 18$ units.

Now, these 18 units of investment will create an aggregate demand of

$Y = \frac{1}{0.2} \times 18$

$Y_t = 90$

As can be seen from above, $Y_t < Y_t^e$, that is, 90 < 99.

Let us now calculate the actual and expected growth rates.

$$g_t^a = \frac{Y_t - Y_{t-1}}{Y_t} = \frac{90-9}{90} = 0\%$$

$$g_t^e = \frac{Y_t^e - Y_{t-1}}{Y_t^e} = \frac{99-9}{99} = 9\%$$

As can be seen now,

$$g_t^a < g_t^e < g_w$$

$$0\% < 9\% < 10\%$$

Thus investors will feel that they expected too much and hence they will reduce investment further in the next period thereby widening the gap between actual and warranted growth.

Check Your Progress 3

1) Why is the equilibrium condition in the Harrod-Domar model called the 'knife-edge equilibrium'?

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2) What happens to an economy when expected growth rate is higher than the warranted growth rate?

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3) Specify the effect of an expected growth rate lower than the warranted growth rate.

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1.7 LIMITATIONS OF HARROD-DOMAR MODEL

Some of the conclusions of the Harrod-Domar model depend on the crucial assumptions made by Harrod and Domar which make this model unrealistic. These are

1) The propensity to save (s) and the capital-output ratio (ϴ) are assumed to be constant. In reality, they are likely to change in the long run and thus modify the requirements for steady growth.

2) The assumption that labour and capital are used in fixed proportions (due to the assumption of constant returns to scale) is untenable. Generally, labour can be substituted for capital and the economy can move smoothly towards a path of steady growth. In fact, unlike Harrod's model, this path is not so unstable that the economy should experience chronic inflation or unemployment if g_t^a does not coincide with g_w.

3) The Harrod-Domar model fails to consider changes in the general price level. Price changes always occur over time and may stabilize otherwise unstable situations. In fact, if allowance is made for price changes and variable proportions in production, then the system may have much stronger stability than what the Harrod's model suggests.

4) The assumption that there are no changes in interest rates is irrelevant to the analysis. In fact, interest rates do change and affect investment. A reduction in interest rates during periods of overproduction can make capital-intensive processes more profitable by increasing the demand for capital, thereby making excess supplies of goods.

5) The Harrod-Domar model ignores the effect of government programmes on economic growth. If, for instance, the government undertakes programmes of development, the Harrod-Domar analysis does not provide us with causal (functional) relationship.

1.8 LET US SUM UP

In this Unit, we discussed the Harrod-Domar Model which is the direct outcome of projection of the short-run Keynesian analysis into the long-run. The Harrod-Domar model shows the importance of saving and investing in an economy. The model was developed independently by Roy F. Harrod and Evsey Domar. According to this model, the growth of an economy is positively related to its saving ratio and negatively related to its capital-output ratio.

The model implies that a higher saving rate allows for more investment in physical capital. This investment can increase the production of goods and services in a country, thereby increasing growth. The capital-output ratio shows how much capital is needed to produce a dollar's worth of output. It reflects the efficiency of using machines. This efficiency means that a lower capital-output ratio leads to higher economic growth since fewer inputs generate higher outputs. The model held a great appeal to the developing world. It is argued that in developing countries, low rates of economic growth and development are linked to low saving rates. This creates a vicious cycle of low investment, low output and low saving. To boost economic growth rate, it is necessary to increase saving either domestically or from abroad. Higher saving creates a virtuous circle of self-sustaining economic growth.

The model suggests that there is no natural reason for an economy to have balanced economic growth. The sustained economic growth requires equality between expected growth rate, actual growth rate and warranted growth rate. This however can only be a coincidence. A slight deviation of actual growth rate from the warranted growth rate will lead the economy drift farther away from the steady state growth path.

The instability in the Harrod-Domar model is due to its rigid assumptions such as the assumptions of a fixed production function, a fixed saving rate, and a fixed capital-output ratio. Despite these limitations, the Harrod-Domar model is important as it makes Keynes' static short-run saving and investment theory a dynamic one.

1.9 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Sustained growth has enormous potential. A sustained growth of even 2%, will double GDP per capita in 35 years.
2) If you think of a circuit diagram with income flowing out of firms as they produce and income flowing back into firms as they sell, you can visualize saving as a leakage from the system.

Check Your Progress 2

1) It is given by the ratio of s to Θ. Refer to Section 1.5.
2) If the economy has to grow steadily, without any disturbances, then the two ratios namely saving rate and capital-output ratio have to be the same over time. If the above holds, then the economy will grow steadily at a given rate of growth $\frac{s}{\Theta}$, which is the ratio of saving rate to capital-output ratio.
3) The equation changes to $\boldsymbol{\frac{s}{\Theta} = g^* + n}$.

Check Your Progress 3

1) A slight deviation of actual growth rate from warranted growth rate leads the economy drift farther away from the steady state growth path.
2) If g_t^e exceeds $g_w = \frac{s}{\Theta}$ than actual growth rate will turn out to be even greater than the already high expected growth rate. Investors would then get a wrong signal. Such a situation will lead to secular inflation because

actual income grows at a faster rate than allowed by the growth in the productive capacity of the economy. The following inequality holds: $g_t^a > g_t^e > g_w = \frac{s}{\theta}$.

3) If $g_t^e < g_w$, than actual growth rate will be even lower than the expected growth rate. In other words, $g_t^a < g_t^e < g_w$. Such a situation will lead to secular depression because actual income grows slowly than what is required for full utilization of the productive capacity of the economy.

UNIT 2 THE SOLOW MODEL*

Structure

2.0 OBJECTIVES

After going through this unit, you will be in a position to

- explain economic growth with the help of neoclassical growth model;
- outline the implications of the assumptions made in the Solow model;
- determine how steady state growth can be achieved in an economy with an exogenous population growth rate and technological progress;
- determine the growth of key variables such as output per worker (means per unit of labour) and capital per worker on the balanced growth path;
- examine the impact of saving rate and population growth on the long run living standards; and
- comment on the golden rule level of capital.

2.1 INTRODUCTION

The limitations of the Harrod-Domar model prompted many economists to think further. Recall from the previous Unit that the warranted growth rate in the

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Harrod-Domar model was given by the ratio of 's' (saving rate) and capital-output ratio 'v'. The razor edge problem came up because s and v are constants, so that their ratio is a constant, and there is no scope for altering this ratio. In real life, however, economies do not face such razor edge problems and policy makers do have certain flexibility. In order to make the Harrod-Domar model more realistic, economists proceeded on two lines. In a major contribution to economic growth theory, Robert M. Solow developed the neoclassical model of economic growth in 1957, for which he was awarded Noble Prize in economic sciences in 1987. Solow has made a huge contribution to our understanding of the factors that determine the rate of economic growth for different countries. Solow extended the Harrod-Domar model by adding labour as a factor of production, and assuming that capital-output ratio is not constant.

The Solow growth model shows how saving, population growth, and technological progress affect the level of an economy's output and its growth over time. It also explains why national income grows, and why some economies grow faster than others.

In this Unit, we begin with the assumptions of the model. Subsequently we derive the steady state growth path. We then introduce the golden rule capital-labour ratio. We also understand how the changes in the savings rate, population and technological progress affect the output per person and capital per person in the steady state. We conclude by discussing implications of the Solow Model for the economies of the world.

2.2 SOURCES OF ECONOMIC GROWTH

Solow considers an aggregate production function which defines the relationship between output (*Y*) and two inputs, viz., capital (*K*) and labour (*L*). In symbols, it is given by

$$Y = EF(K, L) \quad \text{... (2.1)}$$

where E denotes the level of technology.

In equation (2.1), if the levels of inputs (K and L) are constant and the level of technology is the same, output will be constant – there will be no economic growth. For the level of output to grow, either the levels of inputs grow or the level of technology must improve (that means, there should be 'technological progress' or 'productivity growth'), or both. Thus growth of output has two sources, viz., (i) growth in inputs, and (ii) productivity growth. According to Solow, relationship between the rate of output growth, the rates of input growth, and productivity growth is

$$\frac{\Delta Y}{Y} = \frac{\Delta E}{E} + a_K \frac{\Delta K}{K} + a_L \frac{\Delta L}{L} \quad \text{... (2.2)}$$

where

$\frac{\Delta Y}{Y}$ = rate of output growth

$\frac{\Delta E}{E}$ = rate of productivity growth

$\frac{\Delta K}{K}$ = rate of capital input growth

$\frac{\Delta L}{L}$ = rate of labour input growth

a_K = elasticity of output with respect to capital

a_L = elasticity of output with respect to labour

Equation (2.2) is called the 'growth accounting equation'. Growth accounting provides useful information about the sources of growth. It however does not completely explain a country's growth performance. Because growth accounting takes the economy's rates of input growth as given, it cannot explain why capital and labour grow at the rates they do. The growth of capital stock is the result of saving and investment decisions taken by the households and firms, while the growth of labour depends on population growth. By taking capital stock and labour as given, growth accounting presents a static picture. In the next section we take a closer look at the dynamics of economic growth, or how the growth process evolves over time.

2.3 ASSUMPTIONS OF THE SOLOW MODEL

The main assumptions of the Solow model are as follows:

i) The economy is operates under full employment of inputs.

ii) There is no government interference in the functioning of the economy (it is a free market economy).

iii) There is no external trade so that it is a "closed economy".

iv) The economy operates under a *neoclassical production function*. It exhibits constant returns to scale. For example, if all factors of production (K and L in this case) are doubled, output will be exactly doubled. In notations,

$$zY = F(zK, zL) \quad \text{... (2.3)}$$

for any positive number z.

Both capital and labour are essential for production. This production function allows us to analyze all quantities in the economy relative to the size of the labour. If we set $z = \frac{1}{L}$ in equation (2.3) we get,

$$\frac{Y}{L} = F\left(\frac{K}{L}, 1\right) \quad \text{... (2.4)}$$

The output per worker $\frac{Y}{L}$ is a function of capital per worker $\frac{K}{L}$

Let us define $y = \frac{Y}{L}$, output per worker, and $= \frac{K}{L}$, capital per worker.

Thus we can write the production function given at (2.4) as $y = F(k, 1)$, which can be re-formulated as

$$y = f(k) \quad \text{… (2.5)}$$

The production function given as equation (2.5) is assumed to satisfy three conditions, viz., $f(0) = 0,\ f'(k) > 0\ and\ f''(k) < 0$. We interpret these conditions as follows: First, output is zero when capital per worker is zero. Second, marginal product of capital per unit of labour is positive. Third, marginal product of capital per unit of labour increases at a decreasing rate (In other words, marginal product of capital is positive but it declines as capital per unit of labour increases). The production function given at (2.5) above is shown in Fig. 2.1. You can observe that the production function has a positive slope but it becomes flatter as the amount of capital per worker increases, indicating that it exhibits diminishing returns. When k is low, the average worker has very little capital to work with, so an extra unit of capital is very useful and produces a lot of additional output. When k is high, the average worker has a lot of capital already, so an extra unit of capital increases production only slightly.

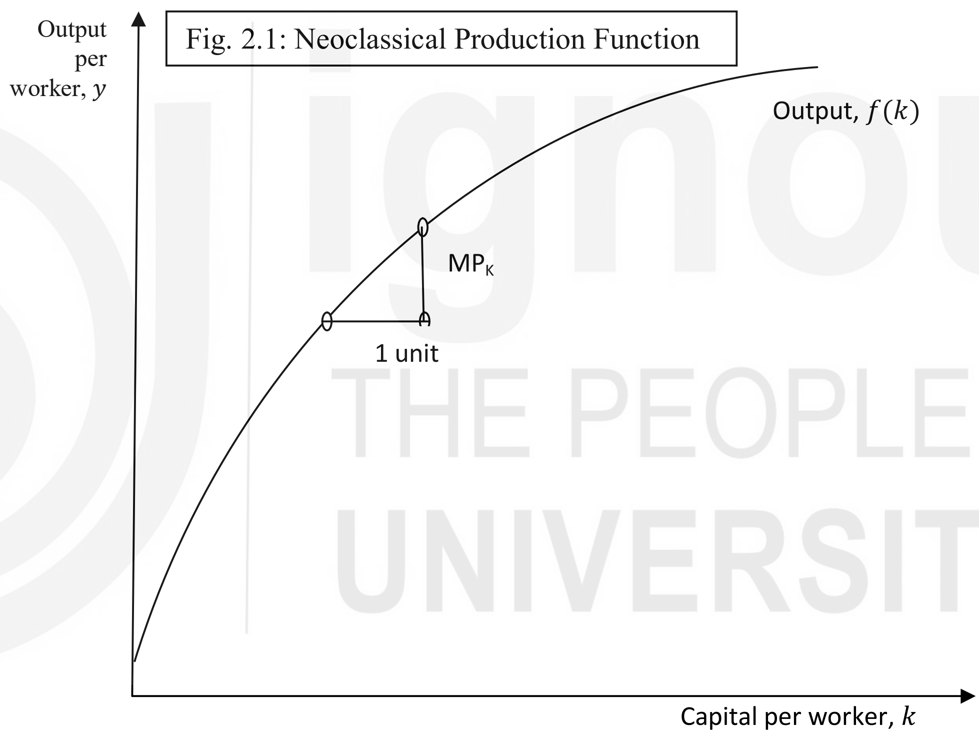

Fig. 2.1 shows that the output per worker depends on capital per worker. The slope of the production function, marginal product of capital – is positive but becomes flatter as k increases, exhibiting diminishing returns to capital per worker.

v) Growth of labour input is exogenously determined (given from outside the model) at a constant rate of n. In notations,

$$n = \frac{1}{L}\frac{dL}{dt}, \quad \frac{dL}{dt} = \dot{L}(t) \qquad \text{... (2.6)}$$

vi) and Technology, E (which we will introduce later) and g respectively. , and $g = \frac{1}{E}\frac{dE}{dt}$ and $\frac{dE}{dt} = \dot{E}(t)$

vii) The Solow model assumes that each year people save – a fraction s of their income and consume a fraction (1–s). The saving rate is fixed. The consumption function can be expressed as

$$c = (1 - s)y \quad ....(2.7)$$

viii) The capital stock depreciates at a constant rate δ every period. Change in capital stock between one period and the next depends on Investment which raises the capital stock and depreciation, which wears out the capital stock.

Change in capital Stock = Investment – Depreciation

$$\dot{K}(t) = I - \delta K(t)\ ,\ \dot{K}(t) = \frac{dK}{dt} \quad ...(2.8)$$

The higher the capital stock, the greater is the amount of depreciation.

Check Your Progress 1

1) State the properties of the production function used in the Solow model.

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2) Describe the growth accounting equation.

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2.4 STEADY STATE GROWTH PATH

The demand for goods comes from consumption and investment.

$$y = c + i \quad ...(2.9)$$

This is the national income identity for a closed economy with no government purchases. The goal is to determine the saving rate which is desirable. Substitute for c from equation (2.7) in equation (2.9)

$$y = (1 - s)y + i \quad ...(2.10)$$

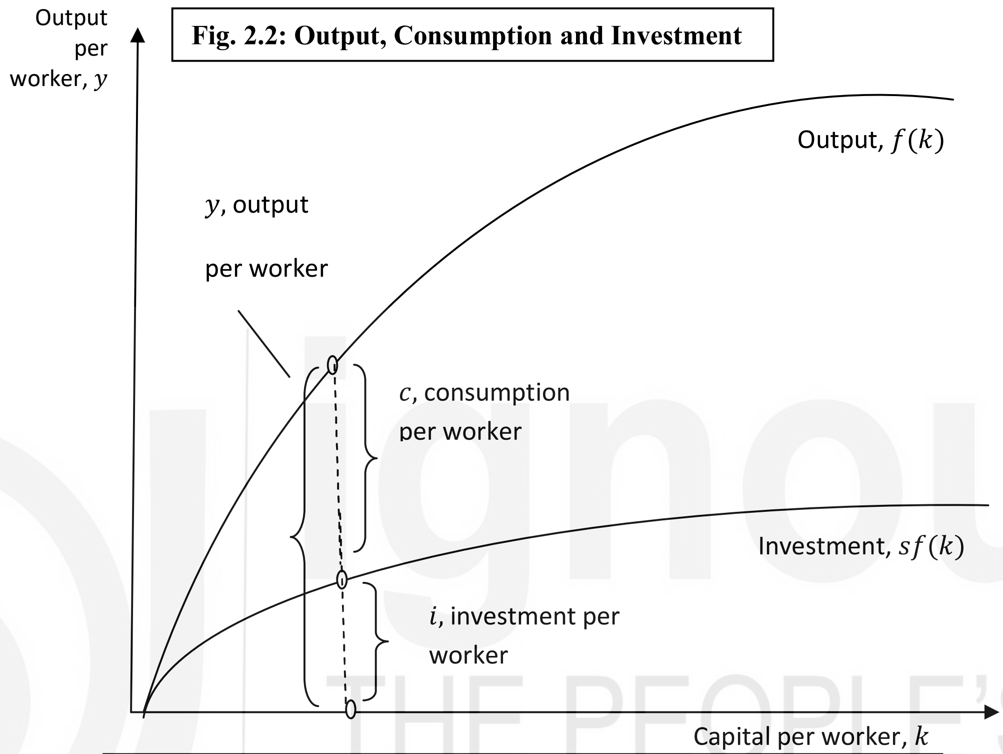
We get, $i = sy$ ...(2.11)

This equation shows that investment equals saving. Thus the rate of saving *s* is also the fraction o output devoted to investment. Let us substitute $y = f(k)$ in equation (2.11). This gives us

$i = sf(k)$ ...(2.12)

Equation (2.12) expresses investment per worker, i, as a function of the capital stock per worker, k. Fig. 2.2 shows for any given capital stock, , the production function $y = f(k)$ determines how much output the economy produces, and the saving rate s determines the allocation of that output between consumption per worker c and investment per worker i .

Fig. 2.2: Output, Consumption and Investment



The saving rate s determines the allocation of output between consumption and investment. For any level of capital k, output is $f(k)$, investment is $sf(k)$, and consumption is $f(k) - sf(k)$.

2.4.1 Dynamics of the Model

We want to determine the behaviour of the economy we have just described. Labour is exogenous and not determined within the model. Thus to characterize the behaviour of the economy we must analyze the behaviour of the other input, capital.

$\mathrm{k} = \frac{\mathrm{K}}{\mathrm{L}}$, we can use the chain rule to find

$$k(\dot{t}) = \frac{K(\dot{t})}{L(t)} - \frac{K(t)}{L(t)^2} * L(\dot{t}) \quad ...(2.13)$$

$$k(\dot{t}) = \frac{K(\dot{t})}{L(t)} - \frac{K(t)}{L(t)} * \frac{L(\dot{t})}{L(t)} \quad ...(2.14)$$

Substitute for $K(\dot{t})$ from equation 2.8 and $\frac{L(\dot{t})}{L(t)} = n$ from equation (2.6) in equation (2.14)

$$\dot{k(t)} = \frac{I - \delta K(t)}{L(t)} - \frac{K(t)}{L(t)} * n \qquad ...(2.15)$$

Substitute $\frac{I}{L} = i \; and \; \frac{K}{L} = k$ in equation (2.15)

$$\dot{k(t)} = i - \delta k - nk \qquad ...(2.16)$$

Substitute i from (2.12) into equation (2.16)

$$\dot{k(t)} = sf(k) - (\delta + n)k \qquad ...(2.17)$$

Equation (2.17) is the key equation of the Solow model. It states that the rate of change of the capital stock per unit of labour is the difference between two terms. The first, $sf(k)$, is the actual investment per unit of labour: output per unit of labour $f(k)$ and the fraction of that output that is invested is s. The second term, $(\delta + n)k$, is break-even investment, the amount of investment that must be done to keep k at its existing level. There are two reasons that some investment is needed to prevent k from falling. First, existing capital is depreciating; this capital must be replaced to keep the capital stock from falling. This is the δk term in equation (2.17). Second the quantity of labour is growing. Since the quantity of labour is growing at rate n, the capital stock must grow at rate n to hold k steady. This is the nk term in equation (2.17). This is the amount of investment necessary to provide new workers, n with capital. The equation shows that population growth reduces the accumulation of capital per worker much the way depreciation does. When actual investment per unit of labour exceeds the investment needed to break-even, k is rising. When actual investment falls short of the break-even investment, k is falling. And when the two are equal, k is constant.

2.4.2 Steady State Level of Capital

A steady state is a situation in which the economy's output per worker, y, consumption per worker c and capital stock per worker k are constant. To explain how the Solow model works, we first examine the characteristics of a steady state and then discuss how economy might attain it. In Fig. 2.3, there is a single capital stock k^* at which the amount of investment equals the amount of depreciation and the amount of investment necessary to provide new workers, n with capital. If the economy finds itself at this level of the capital stock, the capital stock will not change because the two opposing forces acting on it –investment and (depreciation and population growth) – just balance. That is, at k^*, $\dot{k} = 0$, so the capital stock per worker k and output per worker $f(k)$ are steady over time (rather than growing or shrinking). We therefore call k^* the **steady state** level of capital.

The definition of an equilibrium is a positive value of k, denoted by k^* such that $\dot{k} = 0$. This is called the steady state. There is a corresponding value of output per worker , denoted by y^* such that $\dot{y} = 0$. The steady state value of k^* is solved from equation (2.17).

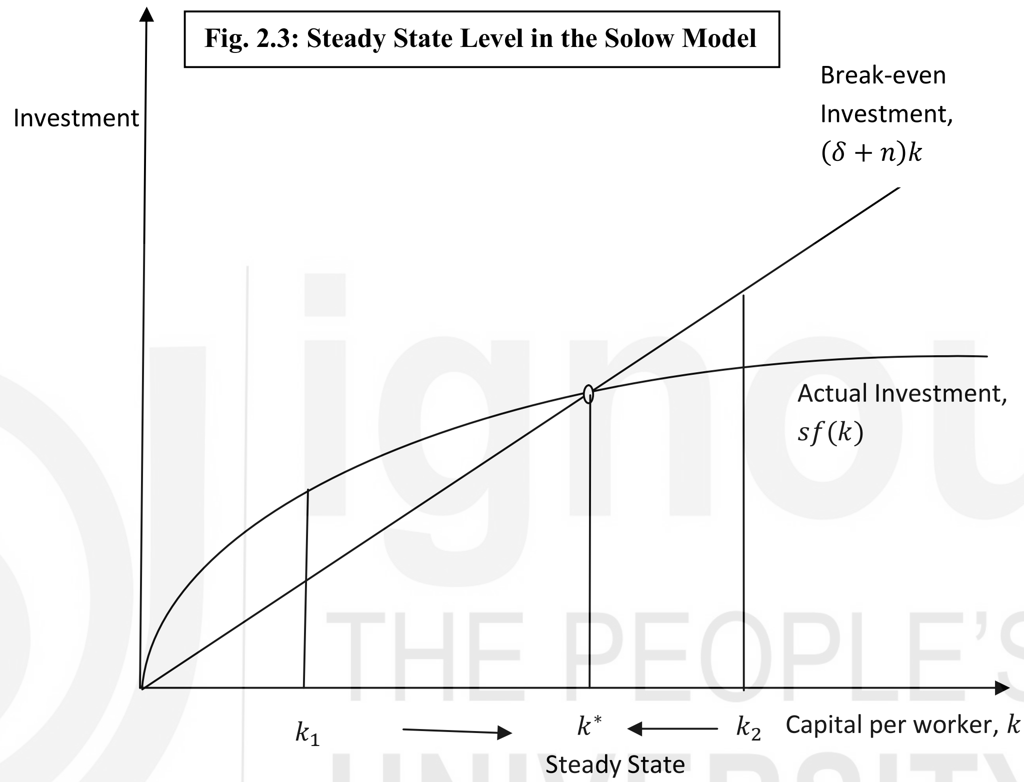
$$0 = sf(k^*) - (\delta + n)k^* \qquad ...(2.18)$$

Then y^* is solved from

$y^* = f(k^*)$

The steady state is significant for two reasons. As we have just seen, an economy at the steady state will stay there. In addition, and just as important, an economy *not* at the steady state will go there. That is, regardless of the level of capital with which the economy begins, it ends up with the steady-state level of capital. In this sense, *the steady state represents the long-run equilibrium of the economy.*

Fig. 2.3: Steady State Level in the Solow Model

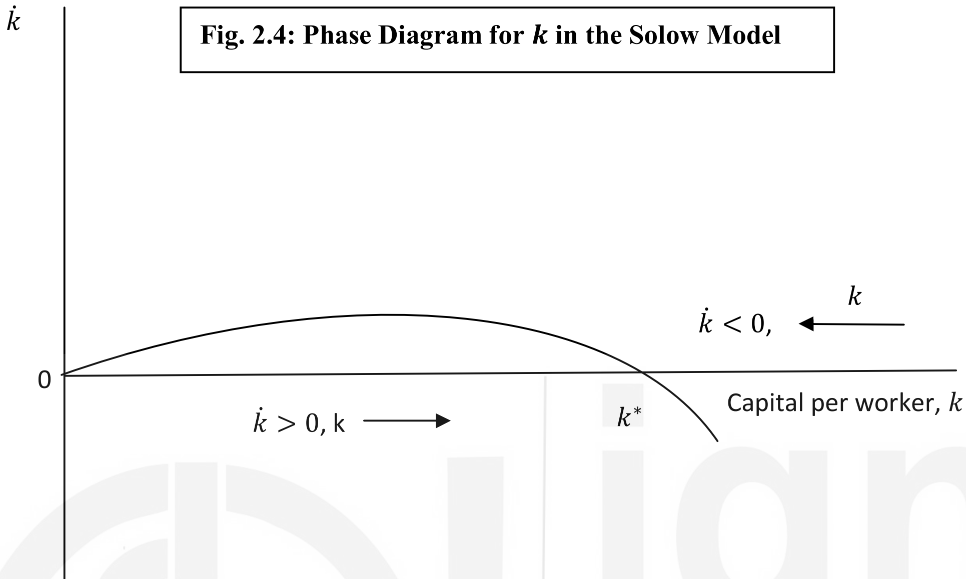


The steady state level of capital k^* is the level at which investment equals break-even investment $(\delta + n)k$. Below k^*, i.e., at k_1 capital stock increases because investment exceeds depreciation and population growth. At k_2, capital stock shrinks. An economy always ends up at the steady state level, k^*.

To see why an economy always ends up at the steady state, suppose that the economy starts with less than the steady-state level of capital, such as level k_1 in Fig. 2.3. In this case, the level of investment exceeds the break-even investment (depreciation and population growth). Over time, the capital stock will rise and will continue to rise, along with output $f(k)$ until it approaches the steady state k^*. Similarly, suppose that the economy starts with more than the steady-state level of capital, such as level k_2. In this case, investment is less than break-even investment; capital is reducing faster than it is being replaced. The capital stock will fall, again approaching the steady-state level. Once the capital stock reaches the steady state, investment equals depreciation and population growth, and there is no pressure for the capital stock to either increase or decrease.

Fig. 2.4 summarizes this information in the form of a phase diagram, which shows $\dot{k}$ as a function of k. If k is initially less than k^*, actual investment exceeds break-even investment and so $\dot{k}$ is positive- that is k is rising. If k exceeds k^*, $\dot{k}$ is negative- that is k is falling. Finally if k equals k^*, $\dot{k}$ is zero. Thus regardless of where k starts, it converges to k^*.

Fig. 2.4: Phase Diagram for *k* in the Solow Model



The phase diagram above shows that the steady-state, which is unique, is also stable: it will be reached in the long-run, i.e., asymptotically. When k is less than steady state level k^*, rate of change in capital per worker is positive, $\dot{k} > 0$ and capital stock per worker k increases to reach k^*.

2.4.3 Balanced Growth Path

In the steady state with population growth, capital per worker and output per worker are constant. Because the number of workers is growing at the rate n, total output and total capital must also be growing at the rate n.

$$y = \frac{Y}{L}, \qquad k = \frac{K}{L}$$

Additionally, in the steady state

$\dot{y} = 0$ and $\dot{k} = 0$

Differentiating k with respect to time

$$\frac{dk}{dt} = \frac{1}{L}\frac{dK}{dt} - \frac{K}{L^2}\frac{dL}{dt} \qquad ...(2.19)$$

$$\frac{dk}{dt} = \frac{K}{L}\left(\frac{1}{K}\frac{dK}{dt} - \frac{1}{L}\frac{dL}{dt}\right) \qquad ...(2.20)$$

$$\frac{1}{k}\frac{dk}{dt} = \frac{1}{K}\frac{dK}{dt} - \frac{1}{L}\frac{dL}{dt} \qquad ...(2.21)$$

In the steady state $\frac{dk}{dt} = 0$, and $\frac{1}{L}\frac{dL}{dt} = n$. Putting this in equation 2.21

$$\frac{1}{K}\frac{dK}{dt} = n \qquad ...(2.22)$$

Similarly differentiating y with respect to time

$$\frac{dy}{dt} = \frac{1}{L}\frac{dY}{dt} - \frac{Y}{L^2}\frac{dL}{dt} \qquad ...(2.23)$$

$$\frac{dy}{dt} = \frac{Y}{L}\left(\frac{1}{Y}\frac{dY}{dt} - \frac{1}{L}\frac{dL}{dt}\right) \qquad ...(2.24)$$

$$\frac{1}{y}\frac{dy}{dt} = \frac{1}{Y}\frac{dY}{dt} - \frac{1}{L}\frac{dL}{dt} \qquad ...(2.25)$$

In the steady state $\frac{dy}{dt} = 0$, and $\frac{1}{L}\frac{dL}{dt} = n$. By substituting these values in equation (2.25), we obtain

$$\frac{1}{Y}\frac{dY}{dt} = n \qquad ...(2.26)$$

In the steady state output and capital stock grows at the rate of population growth, while output per worker and capital per worker remain constant. Thus the Solow model implies that, regardless of its starting point, the economy converges to a balanced growth path- a situation where each variable of the model grows at the rate exogenously given by the population growth.

2.5 THE GOLDEN RULE LEVEL OF CAPITAL ACCUMULATION

If we were to introduce households into the model, their welfare would depend not on output but on consumption: investment is simply an input into production in the future. Thus for many purposes we are likely to be more interested in the behavior of consumption than in the behavior of output. A benevolent policymaker would thus want to choose the steady state with the highest level of consumption. The steady-state value of k that maximizes consumption is called the **Golden Rule level of capital** and is denoted k^*_{Gold}. National income accounts identity

$$y = c + i \qquad …(2.19)$$

Consumption per worker is

$$c = y - i \qquad …(2.20)$$

Because steady state output is $f(k^*)$ and steady state investment is $(\delta + n)k^*$ at the break-even investment k^*; we can express the steady state consumption as

$$c = f(k^*) - (\delta + n)k^* \qquad ...(2.21)$$

This equation shows that an increase in steady-state capital has two opposing effects on steady-state consumption. On the one hand, more capital means more output. On the other hand, more capital also means that more output must be used to replace capital that is wearing out and equip new workers with high level of capital. Fig. 2.5 graphs steady-state output and steady-state break-even investment as a function of the steady-state capital stock.

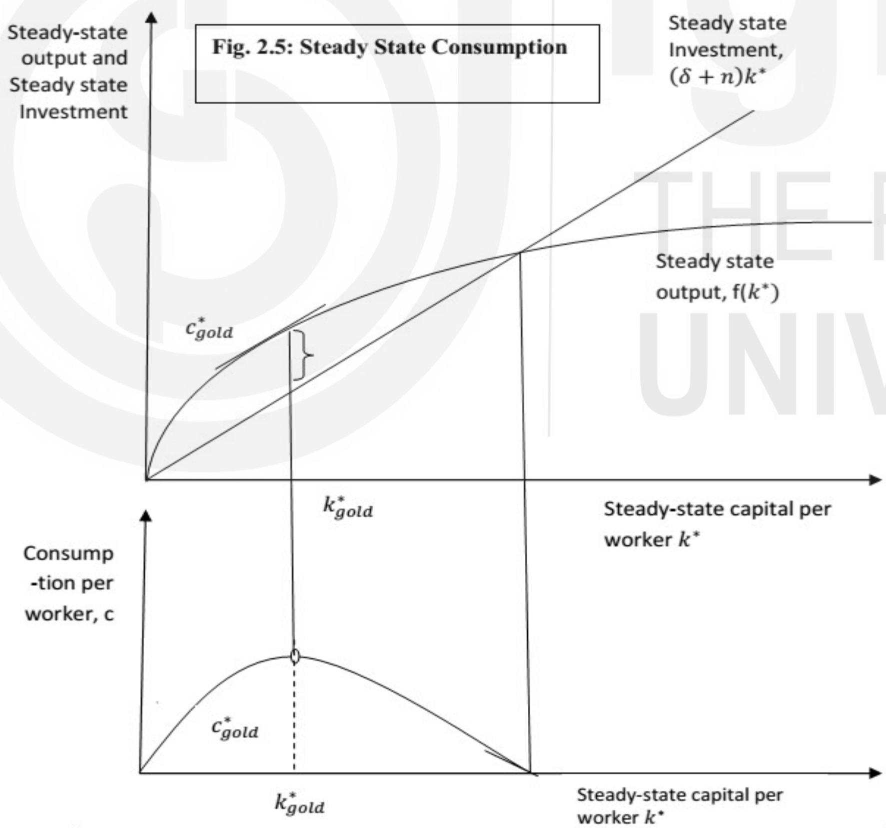
Steady-state consumption is the gap between output and break-even investment. This figure shows that there is one level of the capital stock—the Golden Rule level k^*_{Gold} that maximizes consumption. If the capital stock is below the Golden Rule level, an increase in the capital stock raises output more than break-even investment, so consumption rises. In this case, the production function is steeper than the $(\delta + n)k^*$ line, so the gap between these two curves—which equals consumption—grows as k^*rises. By contrast, if the capital stock is above the Golden Rule level, an increase in the capital stock reduces consumption, because the increase in output is smaller than the increase in break-even investment. In this case, the production function is flatter than the $(\delta + n)k^*$ line, so the gap between the curves—consumption—shrinks as k^*rises. At the Golden Rule level of capital, the production function and the $(\delta + n)k^*$ line have the same slope, and consumption is at its greatest level. Panel b shows consumption per worker depends on the capital per worker. An increase in capital per worker till the Golden rule level raises consumption per worker. A further increase in capital per worker shrinks consumption per worker. The fundamental reason for this outcome is the dimishing marginal productivity of capital-that is, the larger the capital stock already is, the smaller the benefit from expanding the capital stock further. The golden rule level of capital per worker ratio k^*_{Gold} is given by the condition

$$f^{'}(k^*) = \delta + n \quad ...(2.22)$$

$$f^{'}(k^*) - \delta = n \quad ...(2.23)$$

Equation (2.23) implies that marginal productivity of capital, net of depreciation, equals population growth rate at the golden rule level.

Fig. 2.5: Steady State Consumption



Steady state is the difference between output f(k^*) and steady state investment, $(\delta + n)k^*$. Steady state consumption is maximized at the Golden rule steady state. The Golden rule capital stock is denoted k^*_{gold} and the Golden rule consumption is denoted c^*_{gold}.

Check Your Progress 2

1) Explain the dynamics of the Solow Model?

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2) Explain how does an economy always ends up at the steady state?

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3) Explain the condition required to attain Golden rule level of capital.

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2.6 THE FUNDAMENTAL DETERMINANTS OF LONG-RUN LIVING STANDARDS

What determines how well off the average person in an economy will be in the long run? We can use the Solow model to answer this question. Here, we discuss three factors that affect long-run living standards: the saving rate, population growth and productivity growth.

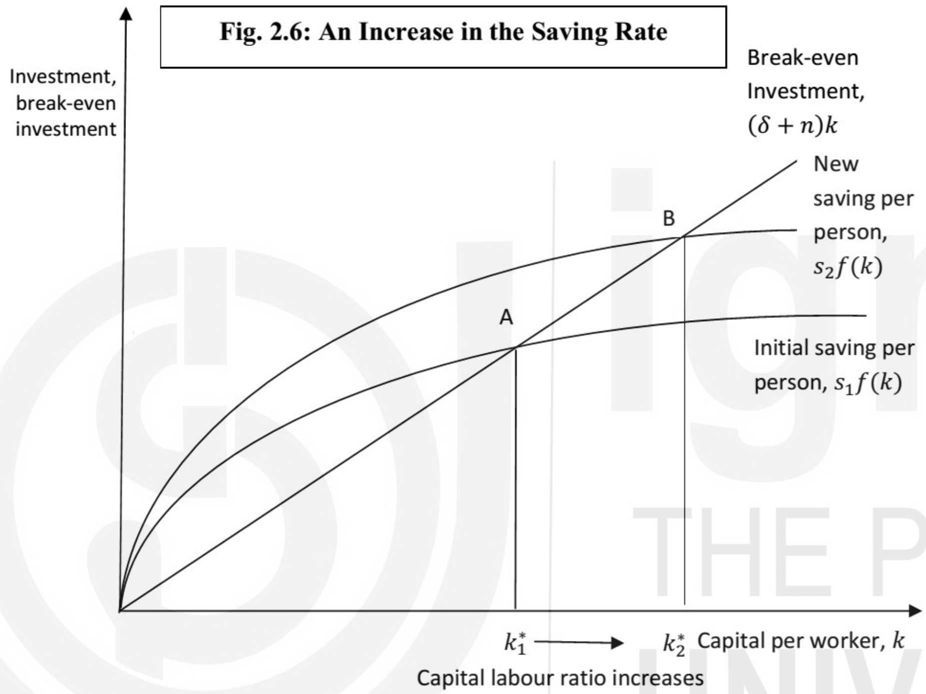
2.6.1 The Impact of Growth in the Saving Rate

According to the Solow model, a higher saving rate implies higher living standards in the long run, as illustrated in Fig. 2.6. Suppose that the initial saving rate is s_1 so that saving per worker is $s_1 f(k)$. The saving curve when saving curve when the saving rate is s_1 is labelled "Initial saving per worker". The initial steady state capital-labour ratio, k_1^* , is the capital labour ratio at which initial saving curve and the break-even investment line cross (point A). Suppose now that the government introduces policies that strengthen the incentives for saving, causing the country's saving rate to rise from s_1 to s_2 . The increased saving rate raises saving at every level of the capital-labour ratio. Graphically, the saving curve shifts upwards fom $s_1 f(k)$ to $s_2 f(k)$. The new steady-state capital-labour ratio, k_2^* , corresponds to the intersection of the new saving curve and the break-even investment line (point B). Because k_2^* is larger than k_1^*, the higher saving rate has increased the steady-state capital-labour ratio. Gradually, this economy will move to the higher steady state capital-labour ratio, as indicated by the arrows on the horizontal axis. In the new steady state, output per worker y, and consumption per worker c will be higher than in the original steady state.

Higher saving rate leads to faster growth in the Solow model, but only temporarily. An increase in the rate of saving raises growth only until the economy reaches the new steady state.

If the economy maintains a high saving rate, it will maintain a large capital stock and a high level of output, but it will not maintain a high growth rate forever. A higher saving rate is said to have a level effect because only the level of output per person- and not its growth rate- is influenced by the saving rate in the steady state.

Fig. 2.6: An Increase in the Saving Rate



An increase in the saving rate s, shifts the saving function upward. At the initial steady state k_1^* , investment now exceeds break-even investment. The capital stock rises until the economy reaches a new steady state k_2^* with more capital and output.

2.6.2 The Impact of Growth in the Population Rate

What is the relationship between population growth and a country's level of development as measured by output, consumption and capital per worker? The Solow model's answer to this question is shown in Fig. 2.7. An initial steady state capital-labour ratio, k_1^* corresponds to the intersection of the break-even investment line and the saving curve at point A. Now suppose that the rate of population growth which is same as the rate of labour force growth increases from an initial level n_1 to n_2 . What will happen to living standards?

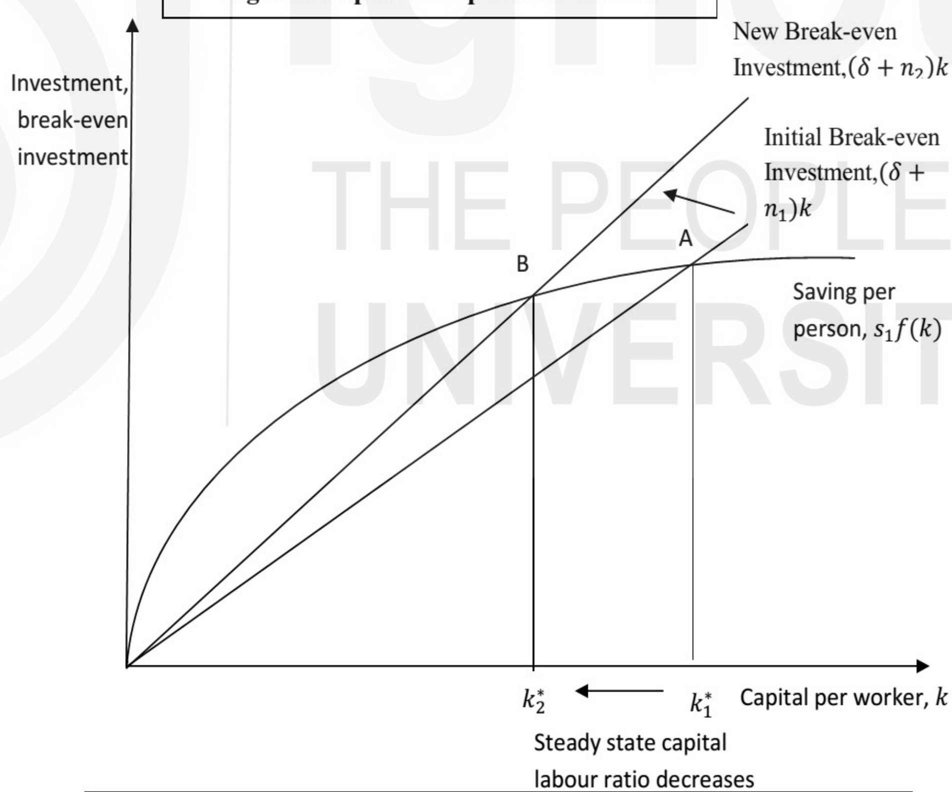
An increase in population growth rate means that workers are entering the labour force more rapidly than before. These new workers must be equipped with capital. Thus, to maintain the same steady-state capital-labour ratio, the amount of investment per current member of workers must rise. Algebraically, the rise in n increases investment per worker from $(\delta + n_1)k$ to $(\delta + n_2)k$.

This increase in the population growth rate causes the break-even investment line to pivot up and to the left (i.e., be steeper), as its slope rises from $(\delta + n_1)$ to $(\delta + n_2)$.

After the pivot of the break-even investment line, the new the steady state is at point B. The new steady-state capital-labour ratio is k_2^*, which is lower than the original capital-labour ratio, k_1^*. Because the new steady state capital-labour ratio is lower, the new steady state output per worker and consumption per worker will be lower as well.

Thus the Solow model implies that increased population growth will lower living standards. The basic problem is that when the work-force is growing rapidly, a large part of current output must be diverted just to providing capital for the new workers to use. This result suggests that policies to control population growth will indeed improve living standards. Notice that a change in the population growth rate, like a change in the saving rate, has a level effect on output per person, but does not affect the steady-state growth rate of output per person.

Fig. 2.7: Impact of Population Growth



Population growth from n_1 to n_2 ,shifts the break-even investment line representing population growth and depreciation upward. The new steady state k_2^* has a lower level of capital per worker than the initial steady state k_1^* .

2.7 TECHNOLOGICAL PROGRESS IN SOLOW MODEL

In the model so far, when the economy reaches its steady state. Output per worker stops growing. To explain persistent growth, we need to introduce technological progress into the model. The model can be modified to include exogenous technological progress, which over time expands society's production capabilities. We now write the production function as

$$\boldsymbol{Y = F(K, L \times E)} \quad ...(2.24)$$

Where E is a new (and somewhat abstract) variable called the **efficiency of labour**. The term $(\boldsymbol{L \times E})$ can be interpreted as measuring the *effective number of workers*. It takes into account the number of actual workers L and the efficiency of each worker E. This new production function states that total output Y depends on the inputs of capital K and effective workers, $\boldsymbol{L \times E}$. We assume that technological progress causes the efficiency of labour $\boldsymbol{E}$ to grow at some constant rate $\boldsymbol{g}$, which is exogenously given.

$$\frac{1}{E}\frac{dE}{dt} = g \quad ...(2.25)$$

This form of technological progress is called *labour augmenting*, and g is called the rate of **labour-augmenting technological progress**. Because the labour force L is growing at rate n, and the efficiency of each unit of labour E is growing at rate g, the effective number of workers $\boldsymbol{L \times E}$ is growing at rate $n + g$. We now analyze the economy in terms of quantities per effective worker. We now let $= \frac{K}{(L\times E)}$, stand for capital per effective worker and $y = \frac{Y}{(L\times E)}$ stand for output per effective worker.. With these definitions, we can again write $y = f(k)$.

$k = \frac{K}{L\times E}$, we can use the chain rule to find

$$\dot{k(t)} = \frac{\dot{K(t)}}{E(t)L(t)} - \frac{K(t)}{E(t)L(t)^2} * \dot{L(t)} - \frac{K(t)}{E(t)^2L(t)} * \dot{E(t)} \quad ...(2.26)$$

$$\dot{k(t)} = \frac{\dot{K(t)}}{E(t)L(t)} - \frac{K(t)}{E(t)L(t)} * \frac{\dot{L(t)}}{L(t)} - \frac{K(t)}{E(t)L(t)} * \frac{\dot{E(t)}}{E(t)} \quad ...(2.27)$$

Let us substitute for $\dot{K(t)}$ from equation 2.8 and $\frac{\dot{L(t)}}{L(t)} = n$ from equation 2.6 and $\frac{\dot{E(t)}}{E(t)} = g$ from equation (2.25) in equation (2.27). This gives us

$$\dot{k(t)} = \frac{I - \delta K(t)}{E(t)L(t)} - \frac{K(t)}{E(t)L(t)} * n - \frac{K(t)}{E(t)L(t)} * g \quad ...(2.28)$$

Substitute $\frac{I}{E\times L} = i$ and $\frac{K}{E\times L} = k$ in equation (2.28)

$$\dot{k(t)} = i - \delta k - nk - gk \quad ...(2.29)$$

By substituting $i = sf(k)$ into equation (2.29), we obtain

$$\dot{k(t)} = sf(k) - (\delta + n + g)k \quad ...(2.30)$$

Equation (2.30) shows the evolution of capital per unit of effective worker, k. The change in capital stock, $\dot{k(t)}$ equals investment $sf(k)$ minus the break-even investment $(\delta + n + g)k$. Break-even investment includes three terms: to keep k constant, δk is needed to replace depreciating capital, nk is needed to provide capital for new workers, and gk is needed to provide capital for the new "effective workers" created by technological progress.

The steady state value of k^* is solved from equation 2.30 by putting $\dot{k(t)} = 0$. We drop the time-subscripts, *t* as output per effective worker and capital per unit of effective labour are constant in the steady state.

Thus, we obtain

$$0 = sf(k^*) - (\delta + n + g)k^* \qquad \text{…(2.31)}$$

Then y^* is solved from

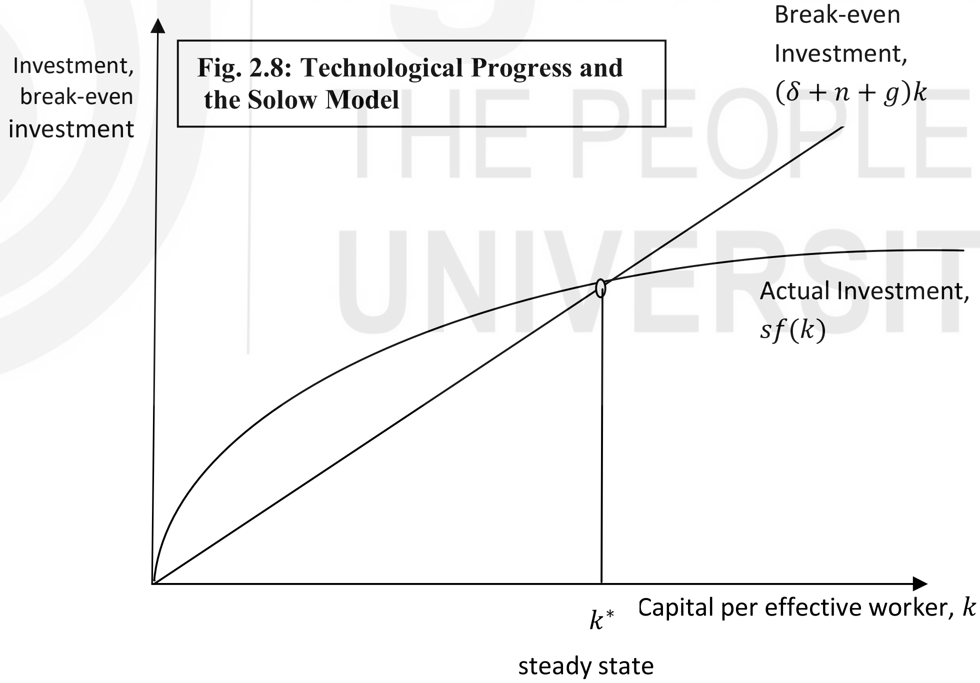
$$y^* = f(k^*)$$

In the steady state, capital per unit of effective labour k^* is given by

$$sf(k^*) = (\delta + n + g)k^* \qquad \text{…(2.32)}$$

As before, in the steady state, investment equals the break-even investment. As shown in Fig. 2.8 there is one level of k, denoted by k^*, at which capital per effective worker and output per effective worker are constant. As before, this steady state represents the long-run equilibrium of the economy.

Fig. 2.8: Technological Progress and the Solow Model



The break-even investment now equals $(\delta + n + g)k$. In the steady state investment $sf(k)$exactly offsets the reductions in k attributable to depreciation, population growth and technological progress.

2.7.1 Balanced Growth Path

By assumption, we have

$$y = \frac{Y}{E \times L}, k = \frac{K}{E \times L}$$

Additionally, in the steady state, $\dot{y} = 0$ and $\dot{k} = 0$.

Differentiating k with respect to time, we obtain

$$\frac{dk}{dt} = \frac{1}{E \times L}\frac{dK}{dt} - \frac{K}{E \times L^2}\frac{dL}{dt} - \frac{K}{E^2 \times L}\frac{dL}{dt} \qquad ...(2.33)$$

$$\frac{dk}{dt} = \frac{K}{E \times L}\left(\frac{1}{K}\frac{dK}{dt} - \frac{1}{L}\frac{dL}{dt} - \frac{1}{E}\frac{dE}{dt}\right) \qquad ...(2.34)$$

$$\frac{1}{k}\frac{dk}{dt} = \frac{1}{K}\frac{dK}{dt} - \frac{1}{L}\frac{dL}{dt} - \frac{1}{E}\frac{dE}{dt} \qquad ...(2.35)$$

In the steady state $\frac{dk}{dt} = 0$, and $\frac{1}{L}\frac{dL}{dt} = n$, $\frac{1}{E}\frac{dE}{dt} = g$. By substituting this in equation (2.35), we obtain

$$\frac{1}{K}\frac{dK}{dt} = n + g \qquad ...(2.36)$$

Similarly, by differentiating y with respect to time, we obtain

$$\frac{dy}{dt} = \frac{1}{E \times L}\frac{dY}{dt} - \frac{Y}{E \times L^2}\frac{dL}{dt} - \frac{Y}{E^2 \times L}\frac{dL}{dt} \qquad ...(2.37)$$

$$\frac{dy}{dt} = \frac{Y}{L}\left(\frac{1}{Y}\frac{dY}{dt} - \frac{1}{L}\frac{dL}{dt} - \frac{1}{E}\frac{dE}{dt}\right) \qquad ...(2.38)$$

$$\frac{1}{y}\frac{dy}{dt} = \frac{1}{Y}\frac{dY}{dt} - \frac{1}{L}\frac{dL}{dt} - \frac{1}{E}\frac{dE}{dt} \qquad ...(2.39)$$

In the steady state, $\frac{dy}{dt} = 0$, and $\frac{1}{L}\frac{dL}{dt} = n$, $\frac{1}{E}\frac{dE}{dt} = g$. Putting this in equation (2.39)

$$\frac{1}{Y}\frac{dY}{dt} = n + g \qquad ...(2.40)$$

From equations (2.36) and (2.40), we now know that in the steady-state, the growth rates of the aggregate capital stock K and aggregate output Y are each in equality with the sum of the technology growth rate and population growth rates.

$$\frac{1}{K}\frac{dK}{dt} = \frac{1}{Y}\frac{dY}{dt} = n + g$$

Alternatively,

$$\frac{1}{K}\frac{dK}{dt} - n = g \text{ , and } \frac{1}{Y}\frac{dY}{dt} - n = g \qquad ...(2.41)$$

The growth rates of the capital stock per labour $\frac{K}{L}$ and of output per labour $\frac{Y}{L}$ are each equal to the technology growth rate. The economy in the long run converges to the balanced growth path. On the balanced growth path, the growth rate of output per worker is solely determined by the growth rate of technological progress.

With the addition of technological progress, our model can finally explain the sustained increases in standards of living that we observe. That is, we have shown that technological progress can lead to sustained growth in output per worker. By contrast, a high rate of saving leads to a high rate of growth only until the steady state is reached. Once the economy is in steady state, the rate of growth of output per worker depends only on the rate of technological progress. *According to the Solow model, only technological progress can explain sustained growth and persistently rising living standards*.

2.7.2 The Golden Rule Level of Capital

The Golden Rule level of capital is now defined as the steady state that maximizes consumption per effective worker. Following the same arguments that we have used before, we can show that steady-state consumption per effective worker is

$$c = f(k^*) - (\delta + n + g)k^* \qquad ...(2.42)$$

Steady state consumption is maximized, if

$$f'(k^*) = \delta + n + g \qquad ...(2.43)$$

$$f'(k^*) - \delta = n + g \qquad ...(2.44)$$

Equation (2.44) implies that, at the Golden Rule level of capital, the net marginal product of capital, $(MPK - \delta)$, equals the rate of growth of total output $(n + g)$.

Table 2.1 The Fundamental Determinants of Long-Run Living Standards

An increase in	Causes long run output, capital and consumption per worker to	Reason
The saving rate, s	Rise	Higher saving allows for more investment and a larger capital stock
The rate of population growth, n	Fall	With higher population growth, more output must be used to equip new workers with capital, leaving less output available for consumption or to increase capital per worker
The rate of technological progress, g	Rise	Higher productivity directly increases output. It also raises savings and the capital stock.

Check Your Progress 3

1) Why an increase in saving rate has only level effect on output per worker?

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2) Explain how an increase in population growth from n_1 to n_2 affect the long-run level of capital and output per worker.

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3) Explain how an increase in the rate of technological progress results in a sustained increase in the standards of living.

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2.8 LET US SUM UP

The Solow model of economic growth is a unique and splendid contribution to economic growth theory. It establishes the stability of the steady-state growth through a very simple and elementary adjustment mechanism. In this unit we have learned that in the Solow growth model saving, population growth and technological progress interact in determining the level and growth of a country's standard of living. In the steady state of the Solow growth model, the growth rate of output per person is equal to the growth rate of capital per worker. Both these growth rates are solely determined by the exogenous rate of technological progress g. The Golden rule (consumption maximizing) steady state is characterized by equality between the net marginal product of capital $(MPK - \delta)$ and the steady state growth rate of total income $(n + g)$. There are two determinants of long run growth in the Solow model- a increase in the saving rate and a fall in the population growth rate. An economy's rate of saving determines the size of its capital stock and thus its level of production. The higher the rate of saving, the higher the stock of capital and the higher the level of output. An economy's rate of population growth is another long-run determinant of the standard of living. According to the Solow model, the higher the rate of population growth, the lower the steady-state levels of capital per worker and output per worker. However, both the changes in saving rate and population rate have level affect output per person but do not affect the steady state growth rate of output per person. It is only the technological progress which can lead to sustained growth in output per worker.

2.9 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) The production function has a positive slope but it becomes flatter as the amount of capital increases, indicating that it exhibits diminishing returns.

2) The relationship between the rate of output growth and the rates of input growth and productivity growth is called the growth accounting equation.

Check Your Progress 2

1) $k(\dot{t}) = sf(k) - (\delta + n)k$,

   The above equation is the key equation of the Solow model. It states that the rate of change of the capital stock per unit of labour is the difference between two terms. The first, $sf(k)$, is the actual investment per unit of labour and the second term, $(\delta + n)k$, is break-even investment.

2) Refer to Fig. 2.3. Suppose that the economy starts with less than the steady-state level of capital, such as level k_1. In this case, the level of investment exceeds the break-even investment (depreciation and population growth). Over time, the capital stock will rise and will continue to rise, along with output $f(k)$ until it approaches the steady state k^*.

3) The golden rule level of capital per worker ratio k^*_{Gold} is given by the condition
   $f^{'}(k^*) = \delta + n$

Check Your Progress 3

1) A higher saving rate is said to have a level effect because only the level of output per person- and not its growth rate- is influenced by the saving rate in the steady state.

2) According to the Solow model, the higher the rate of population growth, the lower the steady-state levels of capital per worker and output per worker. Refer to Fig. 2.7

3) The economy in the long run converges to the balanced growth path. On the balanced growth path, the growth rate of output per worker is solely determined by the growth rate of technological progress.

UNIT 3 ENDOGENOUS GROWTH MODELS*

Structure

3.0 OBJECTIVES

After reading this unit, you will be able to

- explain how increasing returns to scale leads to expansion of output and economic growth;
- identify the reasons behind economic growth of advanced economies in the long run;
- outline the assumptions of the endogenous growth models;
- explain the important features of the AK model;
- determine how saving and investment can lead to persistent growth in the AK model;
- explain the aggregate production function in the Romer model;
- derive the steady state growth rate of output, capital and technology;
- calculate the steady state levels of the key variables such as output-technology ratio and capital-technology ratio on the balanced growth path; and
- examine the impact of a permanent increase in the R&D share in total inputs.

3.1 INTRODUCTION

The Solow model of economic growth (see Unit 2) has proved to be quite useful in our understanding of economic growth. An implication of the Solow model is that there should be convergence in growth across countries in the long run. When capital per labour is low, the returns to capital are very high as a result of which growth rate of the economy is high. When capital per labour increases,

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there is diminishing returns to capital and growth rate slows down. Thus growth rate of poor countries (low capital per labour) should be higher while that of rich countries (high capital per labour) should be lower. In the process there would be convergence in growth rates across countries. Empirically, however, it is observed that rich countries grow richer and poor countries become poorer over time. Even in India, we observe that advanced states have grown faster than backward states over time indicating divergence in economic growth. This type of growth scenario is contrary to the conclusions of the Solow model.

Another limitation of the Solow model, a theoretical one, is its assumption regarding technological progress. According to the Solow model, productivity growth or technological progress is the only source of long run growth of output per capita. Thus an explanation of long run economic growth should include an explanation of productivity growth. The model, however, simply takes the rate of productivity growth as given (exogenous or determined outside the model), instead of explaining how productivity growth is determined. In other words, in the Solow model the determinant of long run growth rate of output per capita is technological progress, which is exogenous to the model.

The Solow model also implied that an increase in rate of savings only has a short run impact on rate of growth and is neutral in its effect on the long run rate of growth. Hence, treating technological change as exogenous, neoclassical theory could not focus on the fundamental forces which determine long run growth of nations.

Endogenous growth theory overcomes these limitations of the Solow model. In this unit we discuss two models, viz., the AK model and the Romer model. We seek an understanding of why the advanced economies of the world, such as the U.S.A. have grown at around 2 per cent per year for the past century. From where does the technological progress that underlines such growth come? Instead of assuming that growth occurs because of unpredictable and exogenous improvements in technology, the endogenous growth theory focuses on understanding the forces underlying technological progress. Hence, main concern of the endogenous growth models is to explain the differences in growth rates among countries and the contributions of different factors to economic growth in these countries. These models go more deeply into the ultimate sources of growth by treating the rate of technological progress or the rate of population growth, or both, as endogenous factors.

3.2 OVERVIEW OF ENDOGENOUS GROWTH MODELS

The endogenous growth theory came into being as a response to the limitations of the Solow model. It tried to explain productivity growth, and hence the growth rate of output, endogenously or within the model. The endogenous growth models reject the Solow model's assumption of exogenous technological progress.

Recall that the neoclassical growth models assumed diminishing returns to inputs so that expansion of output and economic growth cannot take place beyond a level. In order to overcome this restriction to economic growth, endogenous growth models assume that increasing returns to scale is possible.

There are quite a few endogenous growth models in economic literature. We begin by stating the general assumptions of such models model. Subsequently we come to the specifics. The simplest endogenous model is the AK model. We then introduce the aggregate production function in the Romer model. The conditions for balanced growth and steady state levels of key variables, viz., (i) output-technology ratio, and (ii) capital-technology ratio are developed subsequently. We also discuss how the changes in technological progress affect the levels of 'output per person' and 'capital per person' in the steady state. We conclude by discussing implications of the endogenous growth models for the economies of the world.

The main assumptions of endogenous growth models are as follows:

a) There are many firms in a market.

b) Knowledge or technological advance is a non-rival good.

c) There are increasing returns to scale to all factors taken together and constant returns to a single factor (at least for one of the factors).

d) Technological advance comes from things people do. It means that technological advance is based on the creation of new ideas.

e) Many individuals and firms have market power and earn profits from their discoveries. This assumption arises from the fact that there could be increasing returns to scale in production. Increasing returns to scale leads to imperfect competition in the market.

3.3 THE AK MODEL

As pointed out above, the AK model is the simplest endogenous model. It assumes a constant, exogenous, saving rate. It models technological progress with a single parameter (usually denoted by A). It rejects the assumption that the production function exhibit diminishing returns to scale. Our simple endogenous growth model is based on the aggregate production function

$$\mathbf{Y = AK} \qquad \text{... (3.1)}$$

where Y is aggregate output, K is aggregate capital stock and A is a positive constant measuring the amount of output produced for each unit of K. Thus in (3.1) we assume that Y is a constant proportion of K.

According to the production function in equation (3.1), each additional unit of capital increases output by A units, regardless of how many units of capital are used in production. Because the marginal product of capital, equal to A, does not depend on the size of the capital stock K, the production function in equation (3.1) does not imply diminishing marginal productivity of capital. The assumption that the marginal productivity is constant, rather than diminishing, is a key departure from the Solow model.

Endogenous growth theorists have provided a number of reasons to explain why, for the economy as a whole, the marginal productivity of capital may not be diminishing. One explanation emphasizes the role of human capital. In economics, the term human capital means knowledge, skills, and training of individuals. As economies accumulate capital and become richer, they devote more resources towards "investment in people", through improved nutrition, schooling, health care, and on the job training. This investment in people increases the countries' human capital, which in turn raises productivity. If the physical capital stock increases while the stock of human capital stock remains fixed, there will be diminishing marginal productivity of physical capital, as each unit of physical capital effectively works with a smaller amount of human capital. Endogenous growth theory argues that, as an economy's physical capital stock increases, its human capital stock tends to increase in the same proportion. Thus, when the physical capital stock increases, each unit of physical capital effectively works with the same amount of human capital, so the marginal productivity of capital need not decrease.

A second rationale for constant marginal productivity of capital is based on the observation that, in a growing economy, firms have incentives to undertake research and development (R&D) activities. This R&D increases the technical know-how and results in productivity gains. Such gains offset any tendency for the marginal productivity of capital to decline.

In the above, we examined why a production function like equation (3.1) might be a reasonable description of the economy as a whole. We took into account factors such as increased human capital and R&D. Let us now find out the implications of equation (3.1). As in the Solow model, let us assume that national saving, S, is a constant fraction 's' of aggregate output, AK (since Y=AK).

Thus, $S = sY = sAK$. As you from introductory macroeconomics, investment must equal saving in a closed economy. Recall that gross investment equals net investment (the net increase in the capital stock) plus depreciation (*dK*), that is,

$$\boldsymbol{I = \Delta K + dK} \quad \textbf{... (3.2)}$$

Therefore, setting investment equal to saving, we have

$$\boldsymbol{\Delta K + dK = sAK} \quad \textbf{...(3.3)}$$

The growth rate of capital stock thus is

$$\boldsymbol{\frac{\Delta K}{K} = (sA - d)} \quad \textbf{...(3.4)}$$

Because output is proportional to the capital stock (see equation (3.1)), the growth rate of output $\frac{\Delta Y}{Y}$ equals the growth rate of the capital stock $\frac{\Delta K}{K}$. Therefore equation (3.4) implies that

$$\frac{\Delta Y}{Y} = sA - d \qquad ...(3.5)$$

From equation (3.5) we can find out the factors determine the growth rate of output $\frac{\Delta Y}{Y}$. Notice that, as long as $sA > d$, the economy's income grows forever, even without the assumption of exogenous technological progress. The growth rate of output in equation (3.5) depends on the saving rate (s). This is in sharp contrast to the Solow model. Recall that in Solow model the saving rate does not affect the long-run growth rate of the economy. In the AK model, however, the saving rate affects long-run growth of the economy. This result is more realistic because higher rates of saving and capital formation encourages human capital formation and provides incentives for R&D. The resulting increases in productivity help to spur long run growth. In summary, in comparison to the Solow model, places greater emphasis on the saving, human capital formation and R&D as sources of long run growth.

Thus a simple change in the production function can alter dramatically the predictions about economic growth. In the Solow model, saving leads to growth temporarily, but diminishing returns to capital eventually force the economy to approach a steady state in which growth depends only on exogenous technological progress. By, contrast, in this endogenous growth model, saving and investment can lead to persistent growth.

Although endogenous growth theory remains in a developmental stage, the approach appears promising in at least two dimensions. First, this theory attempts to explain, rather than assumes, the economy's rate of productivity growth. Second it shows how the long run growth rate of output may depend on factors, such as the country's saving rate that can be affected by government policies.

Check Your Progress 1

1) State the unique assumption of the AK model?

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2) What determines the growth rate of output in the AK model?

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3.4 THE ROMER GROWTH MODEL

Romer's model of Endogenous Technological Change of 1990 identifies a research sector specializing in the production of ideas. This sector invokes human capital along with the existing stock of knowledge to produce ideas or new knowledge. To Romer, ideas are more important than natural resources. He cites the example of Japan which has very few natural resources but it was open to new western ideas and technology.

Romer describes the aggregate production function as

$$\boldsymbol{Y = L_Y^{1-\alpha}\,(x_1^{\alpha} + x_2^{\alpha} + \cdots . + x_A^{\alpha})} \qquad \text{…(3.6)}$$

$$\boldsymbol{Y = L_Y^{1-\alpha}\sum_{i=1}^{A} x_i^{\alpha}} \qquad \text{…(3.7)}$$

Where L_Y is the number of workers producing output and the $x_{i\,s}^{'}$ are different types of capital goods. A large number of perfectly competitive firms combine labor and capital to produce a homogenous output good, Y. Output Y is produced using labor L_Y and a number of different capital goods, x_i which we call intermediate goods. At any point in time, A measures the number of capital goods that are available to be used in the final goods sector. Inventions or ideas in the model correspond to the creation of new capital goods that can be used by the final goods sector to produce output.

If A was fixed, the pattern of diminishing returns to each of the separate capital goods would mean that growth would eventually taper off to zero. However, in the Romer model, A is not fixed. Instead there are L_A workers engaged in R&D and this leads to invention of new capital goods. When we recognize that ideas (A) are also an input into the production function, then there are increasing returns.

We can define the aggregate capital stock as

$$\boldsymbol{K = \sum_{i=1}^{A} x_i} \qquad \text{…(3.8)}$$

Again we will treat the savings rate as exogenous and assume

$$\boldsymbol{\dot{K} = s_K Y - dK} \qquad \text{…(3.9)}$$

Capital accumulates as people in the economy forgo consumption at some given rate, s_K and depreciates at the exogenous rate, d. One observation that simplifies the analysis is the fact that all of the capital goods play an identical role in the production process. For, this reason, we can assume that the demand from producers for each of these capital goods is the same, implying that

$$\boldsymbol{x_i = \bar{x}\ , i = 1, 2 \ldots\ldots\ldots , A} \qquad \text{…(3.10)}$$

This means that the production function can be written as

$$\boldsymbol{Y = AL_Y^{1-\alpha}\,\bar{x}^{\alpha}} \qquad \text{…(3.11)}$$

Note now that $\boldsymbol{K = A\bar{x}}$ from equation 3.8 and equation 3.10

$$\boldsymbol{\bar{x} = \frac{K}{A}} \qquad \text{…(3.12)}$$

So output can be expressed as

$$Y = AL_Y^{1-\propto}\left(\frac{K}{A}\right)^{\propto} = (AL_Y)^{1-\propto}K^{\propto} \qquad \ldots(3.13)$$

The aggregate production function in the Romer model describes how the capital stock, K, and labor L_Y , combine to produce output, Y, using the stock of ideas, A. $\propto$ is a parameter between 0 and 1. For a given level of technology, the production function in equation 3.13 exhibits constant returns to scale in K and L_Y . However when we recognize that ideas (A) are also an input into the production function, then there are increasing returns.

The total labor supply in the economy is used in two activities. L_Y workers are used to produce output and L_A workers are engaged in R&D and this leads to the invention of new capital goods.

$$L_Y + L_A = L \qquad \ldots(3.14)$$

We assume a constant fraction of labor force is engaged in R&D to produce new ideas

$$\frac{L_A}{L} = s_A \qquad \ldots(3.15)$$

The remaining fraction of workers is used to produce output

$$\frac{L_Y}{L} = 1 - s_A = s_Y \qquad \ldots(3.16)$$

Labor which is equivalent to the population, grows exponentially at some constant and exogenous rate n.

$$\frac{\dot{L}}{L} = n \qquad \ldots(3.17)$$

$A(t)$ is the stock of knowledge or the number of ideas that have been invented over the course of history until time t. $\dot{A}$ is the number of new ideas produced at any given point in time. $\dot{A}$ is described using a production function for the change in the number of capital goods, ideas:

$$\dot{A} = \bar{\gamma}\, L_A^{\lambda} \qquad \ldots(3.18)$$

$\dot{A}$ depends positively on the number of researchers attempting to discover new ideas, L_A. λ is an index of how slowly diminishing marginal productivity sets in for researchers. For example duplication of effort is more likely when there are more persons engaged in research. λ is some parameter between 0 and 1. $\bar{\gamma}$ is the rate at which they discover new ideas. This rate of discovery $\bar{\gamma}$ would depend on the stock of ideas that have already been invented, A.

$$\bar{\gamma} = \gamma A^{\theta} \qquad \ldots(3.19)$$

If $\theta > 0$, this rate of discovery would be an increasing function of A that is the invention of ideas in the past raises the productivity of researchers in the present. If $\theta < 0$, this rate of discovery would be a decreasing function of A, and corresponds to the fishing out case in which the fish become harder to catch over time. Finally, $\theta = 0$ implies that the productivity of research is independent of the stock of knowledge.

Rewrite the general production function for ideas from equations (3.18) and (3.19)

$$\dot{A} = \gamma L_A^{\lambda} A^{\theta} \quad \text{…(3.20)}$$

We assume that $\theta < 1$. This effect stems from the "giant shoulders" effect. For instance, invention of new software must have relied upon previous invention of relevant computer hardware. We also assume $\lambda < 1$ may reflect an externality associated with duplication: some of the ideas created by an individual researcher may not be new to the economy

Check Your Progress 2

1) Explain the aggregate production function of the Romer Model?

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2) Explain what do Θ *and* λ represent in the production function of ideas?

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3) Explain the condition required to attain Golden rule level of capital.

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3.5 THE STEADY STATE GROWTH IN THE ROMER MODEL

This economy converges to a steady state growth path in which capital and output grow at the same rate. So, we can derive the steady state growth rate as follows. Re-write the production function after substituting for L_Y from equation (3.16) as

$$Y = (A s_Y L)^{1-\alpha} K^{\alpha} \quad \text{…(3.21)}$$

Taking logs and derivatives of both sides of equation (3.21)

$$\frac{\dot{Y}}{Y} = (\mathbf{1} - \boldsymbol{\alpha})\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \boldsymbol{\alpha}\left(\frac{\dot{K}}{K}\right) \quad \dots(3.22)$$

Now use the fact that the steady state growth rates of capital and output are the same to derive that this steady state growth rate is given by

$$\left(\frac{\dot{Y}}{Y}\right)^* = (\mathbf{1} - \boldsymbol{\alpha})\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \boldsymbol{\alpha}\left(\frac{\dot{Y}}{Y}\right) \quad \dots(3.23)$$

Now $\frac{\dot{s_Y}}{s_Y} = \mathbf{0}$ …(3.24)

That is the share of labor allocated to the non-research sector cannot be changing along the steady state path otherwise the fraction of researchers would eventually go to zero or become greater than 1, which would be infeasible. So we have,

$$\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right)^* = \frac{\dot{A}}{A} \quad \dots(3.25)$$

The steady state growth rate of output per worker equals the steady state growth rate of A. Let lower case letters denote per capita variables and let g_X denote the growth rate of variable X. Then

$$\boldsymbol{g_Y = g_K = g_A} \quad \dots(3.26)$$

That is per capita output, capital labor ratio and stock of ideas must all grow at the same rate along a balanced growth path. If there is no technological progress in the model, then there is no growth. Therefore we must work out this rate of technological growth along a balanced growth path.

3.6 DERIVING THE STEADY STAE GROWTH RATE IN THE ROMER MODEL

The big difference relative to the Solow model is that the A term is determined within the model as opposed to evolving at some fixed rate unrelated to the actions of the agents in the model economy. To derive the steady-state growth rate in this model, note that the growth rate of the number of capital goods is

$$\frac{\dot{A}}{A} = \frac{\gamma (s_A L)^{\lambda}}{A^{1-\theta}} \quad \dots(3.27)$$

The steady state of this economy implies that the parameter is A growing at a constant rate. This can only be the case if the growth rate of the right hand side of equation (3.27) is zero.

Taking log and derivatives of both sides of equation (3.27), we get

$$0 = \boldsymbol{\lambda}\left(\frac{\dot{s_A}}{s_A} + \frac{\dot{L}}{L}\right) - (\mathbf{1} - \boldsymbol{\Theta})\frac{\dot{A}}{A} \quad \dots(3.28)$$

Again in the steady state, the growth rate of the fraction of researchers $\frac{\dot{s_A}}{s_A}$ must be zero. So along the model's steady state growth path, the growth rate of the number of capital goods is

$$\left(\frac{\dot{A}}{A}\right)^* = \frac{\lambda}{1-\theta}\frac{\dot{L}}{L} \quad \dots(3.29)$$

Substituting for $\frac{\dot{L}}{L} = n$ from equation (3.17), we get

$$\boldsymbol{g_Y = g_K = g_A = \left(\frac{\dot{A}}{A}\right)^* = \frac{\lambda}{1-\theta} n} \quad \text{...(3.30)}$$

The long run growth rate of output per worker in this model depends positively on three factors.

a) The parameter λ, which describes the extent to which diminishing marginal productivity sets in as we add researchers. We may refer to the externality associated with λ as stepping on toes effect.

b) The strength of the "standing on shoulders" effect, θ. The more past inventions help to boost the rate of current inventions, the faster the growth rate will be. It reflects a positive knowledge spill-over in research. We may refer to the externality associated with θ as standing on shoulders effect.

c) The growth rate of the number of workers, n. The higher is n, the faster the economy adds researchers. This may seem like a somewhat unusual assumption but it holds well if one takes a very long view of world economic history. Prior to the industrial revolution, growth rates of population and GDP per capita were very low. The past 200 years have seen both population growth and economic growth rates increases.

3.7 THE STEADY STATE LEVEL OF OUTPUT TECHNOLOGY RATIO

Just as with our discussion of the Solow model, we can decompose output per worker into a capital-output ratio component and a Total factor productivity component. In other words one can re-arrange equation (3.13) by substituting $\boldsymbol{L_Y = (1 - s_A)L}$

$$\boldsymbol{Y = (A\,(1 - s_A)L)^{1-\alpha} K^{\alpha}} \quad \text{...(3.31)}$$

From equation (3.9), $\dot{K} = s_K Y - dK$

$$\boldsymbol{\frac{\dot{K}}{K} = s_K \frac{Y}{K} - d} \quad \text{...(3.32)}$$

WE find that equation (3.32) is the capital accumulation equation.

Let us divide both sides of equation (3.31) by L, so that we get output per capita

$$\boldsymbol{y = \frac{Y}{L} = A^{1-\alpha}(1 - s_A)^{1-\alpha}\left(\frac{K}{L}\right)^{\alpha}} \quad \text{...(3.33)}$$

Let us substitute $\boldsymbol{k = \frac{K}{L}}$ in equation (3.33).

$$\boldsymbol{y = A^{1-\alpha}(1 - s_A)^{1-\alpha} k^{\alpha}} \quad \text{...(3.34)}$$

Let us divide both sides of equation (3.34) by A.

This gives us

$$\boldsymbol{\frac{y}{A} = (1 - s_A)^{1-\alpha}\left(\frac{k}{A}\right)^{\alpha}} \quad \text{...(3.35)}$$

If we substitute $\bar{y} = \frac{y}{A}$ and $\bar{k} = \frac{k}{A}$ in equation (3.35) we get

$$\boldsymbol{\bar{y} = (1 - s_A)^{1-\alpha} \bar{k}^{\alpha}} \quad \ldots(3.36)$$

where $\bar{y} = \frac{y}{A} = \frac{Y}{AL}$ is output per worker to technology or output technology ratio and $\bar{k} = \frac{k}{A} = \frac{K}{AL}$ is capital per worker to technology or capital technology ratio.

If we rewrite the capital accumulation equation (3.32) in terms of $\bar{k}$, we obtain the capital technology ratio

$$\boldsymbol{\frac{\dot{\bar{k}}}{\bar{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}} \quad \ldots(3.37)$$

Let us substitute for $\frac{\dot{K}}{K} = s_K \frac{Y}{K} - d$, $\frac{\dot{A}}{A} = g_A$, and $\frac{\dot{L}}{L} = n$ in equation (3.37). This gives us

$$\boldsymbol{\frac{\dot{\bar{k}}}{\bar{k}} = s_K \frac{Y}{K} - d - g_A - n} \quad \ldots(3.38)$$

$$\boldsymbol{\frac{\dot{\bar{k}}}{\bar{k}} = s_K \left(\frac{Y/_{AL}}{K/_{AL}}\right) - d - g_A - n} \quad \ldots(3.39)$$

$$\boldsymbol{\frac{\dot{\bar{k}}}{\bar{k}} = s_K \frac{\bar{y}}{\bar{k}} - d - g_A - n} \quad \ldots(3.40)$$

$$\boldsymbol{\dot{\bar{k}} = s_K \bar{y} - (d + g_A + n)\bar{k}} \quad \ldots(3.41)$$

Solving for steady state output technology ratio is determined by the production function and the condition that in the steady state $\dot{\bar{k}} = 0$. We solve for $\bar{k}^*$ by putting $\dot{\bar{k}} = 0$ in equation 3.41

$$\boldsymbol{0 = s_K \bar{y} - (d + g_A + n)\bar{k}} \quad \ldots(3.42)$$

$$0 = \boldsymbol{s_K \left[(1 - s_A)^{1-\alpha} \bar{k}^{\alpha}\right] - (d + g_A + n)\bar{k}} \quad \ldots(3.43)$$

$$\boldsymbol{\bar{k}^{1-\alpha} = (1 - s_A)^{1-\alpha} \frac{s_K}{d + g_A + n}} \quad \ldots(3.44)$$

$$\boldsymbol{\bar{k}^* = (1 - s_A) \left(\frac{s_K}{d + g_A + n}\right)^{\frac{1}{1-\alpha}}} \quad \ldots(3.45)$$

Substituting $\bar{k}^*$ from equation (3.45) into the production function in equation (3.36), we get

$$\boldsymbol{\bar{y}^* = (1 - s_A)^{1-\alpha} \left(\frac{s_K}{d + g_A + n}\right)^{\frac{\alpha}{1-\alpha}} (1 - s_A)^{\alpha}} \quad \ldots(3.46)$$

Hence steady state output technology ratio is given by

$$\boldsymbol{\bar{y}^* = (1 - s_A) \left(\frac{s_K}{d + g_A + n}\right)^{\frac{\alpha}{1-\alpha}}} \quad \ldots(3.47)$$

3.8 PERMANENT INCREASE IN THE SHARE OF R&D

In this section we try to answer what happens to the advanced economies of the world if the share of population searching for new ideas increases permanently? To simplify things slightly, let's assume that $\lambda = 1$ *and* $\Theta = 0$. We can re write equation (3.27) as

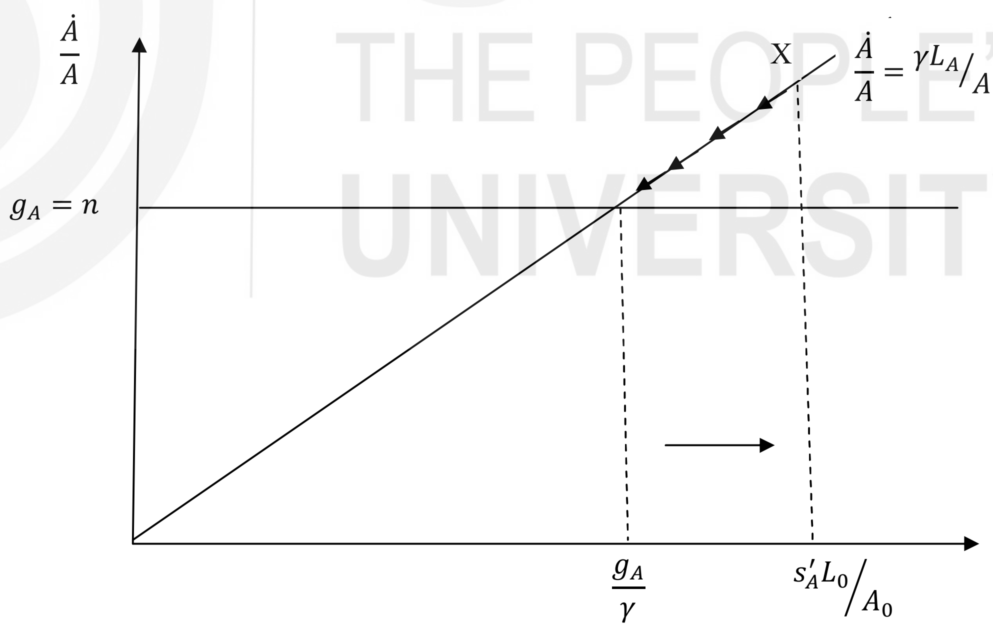
$$\frac{\dot{A}}{A} = \frac{\gamma(s_A L)}{A} \qquad ...(3.48)$$

Fig. 3.1 shows what happens to technological progress when s_A increases permanently to $s_A{}'$, assuming that economy begins in a steady state. In steady state, the economy grows along a balanced growth path at the rate of technological progress, g_A, which happens to equal the rate of population growth, n under our simplifying assumptions

$$g_A = \frac{\dot{A}}{A} = \frac{\gamma(s_A L)}{A} \qquad ...(3.49)$$

$$\frac{g_A}{\gamma} = \frac{\dot{A}}{A} = \frac{L_A}{A} \qquad ...(3.50)$$

Fig. 3.1: Technological Progress: An Increase in the R&D Share

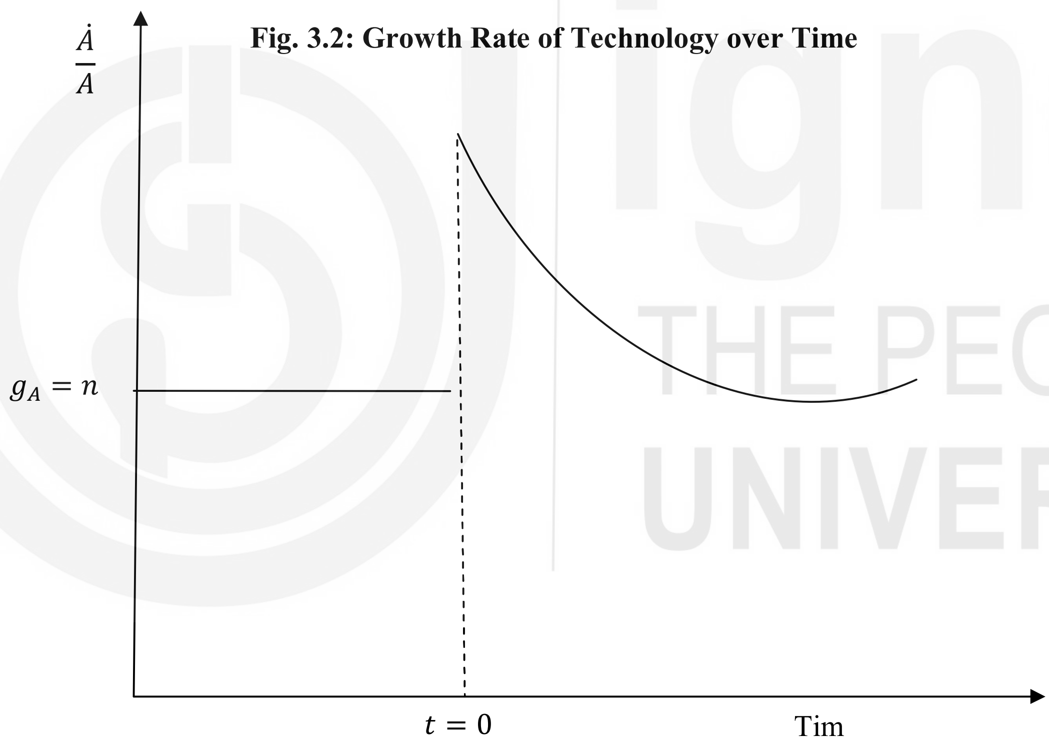


In steady state, the economy grows along a balanced growth path at $g_A = n$. With an increase in s_A *at* $t = 0$, the number of researchers increases and the ratio of $\frac{L_A}{A}$ jumps to a higher level X. At X, technological progress $\frac{\dot{A}}{A}$ exceeds population growth n, so the ratio $\frac{L_A}{A}$ declines over time, as indicated by the arrows.

From equation (3.50) the ratio of $\frac{L_A}{A}$ is therefore equal to $\frac{g_A}{\gamma}$. In Fig. 3.1, suppose the increase in s_A occurs at time $t = 0$. With a population of L_0, the number of researchers increases as L_A increases, so that the ratio $\frac{L_A}{A}$ jumps to a higher level. The additional researchers produce an increased number of new ideas, so the growth rate of technology is also higher at this point. This situation corresponds to the point labelled "X" in Fig. 3.1. At X, technological progress $\frac{\dot{A}}{A}$ exceeds population growth n, so the ratio $\frac{L_A}{A}$ declines over time, as indicated by the arrows.

As this ratio declines, the rate of technological change gradually falls also, until the economy returns to the balanced growth path where $g_A = n$. Therefore, a permanent increase in the share of the population devoted to research raises the rate of technological progress temporarily, but not in the long run. This behaviour is depicted in Fig. 3.2.

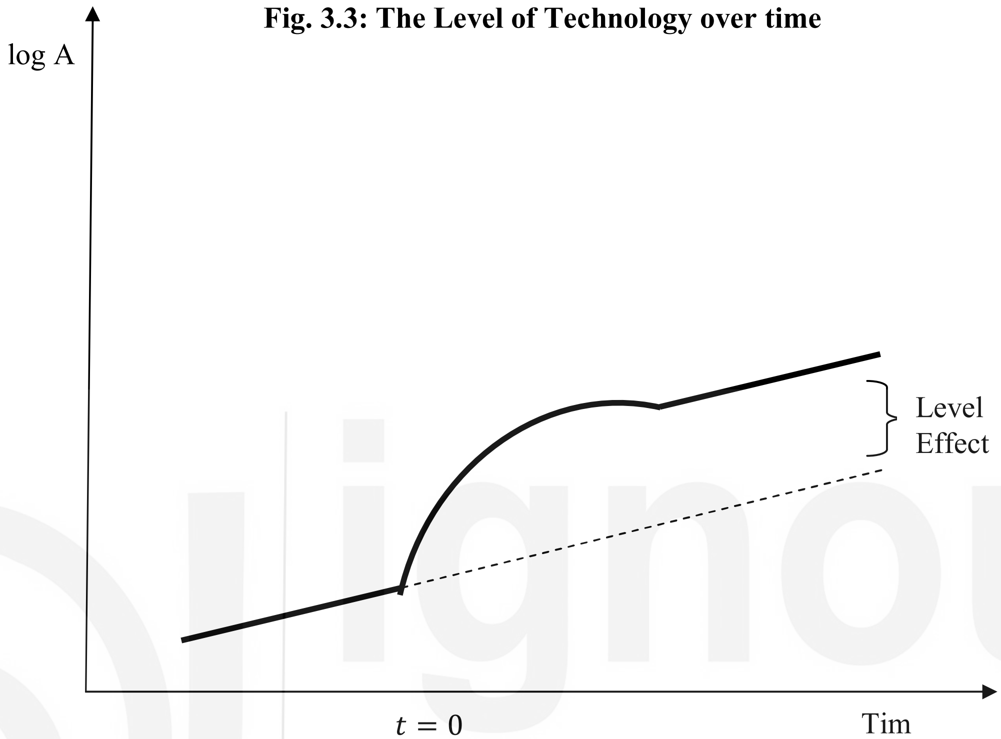
Fig. 3.2: Growth Rate of Technology over Time



The ratio of $\frac{L_A}{A}$ continues to decline until the economy returns to the balanced growth path where $g_A = n$. Rate of technological progress raises only temporarily.

What happens to the level of technology in this economy? Fig. 3.3 answers this question. The level of technology is growing along a balanced growth path at rate g_A until time $t = 0$. At this time, the growth rate increases and the level of technology rises faster than before. Over time, however, the growth rate falls until it returns to g_A. The level of technology is permanently higher as a result of the permanent increase in R&D. Thus a permanent increase in the share of population devoted to research has only a level effect on technology. The level of technology is permanently higher as a result of the permanent increase in R&D. The long-run growth rate of the model returns to the balanced growth path after increasing temporarily.

Fig. 3.3: The Level of Technology over time



The level of technology is growing along a balanced growth path at rate g_A until time $t = 0$. At this time, the growth rate increases and the level of technology rises faster than before. Over time, however, the growth rate falls until it returns to g_A. The level of technology is permanently higher as a result of the permanent increase in R&D.

Check Your Progress 3

1) What is the steady state growth rate of output per worker in the Romer model?

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2) Describe the determinants of the long run growh rate of output per worker?

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3) Derive the equation for the level of steady state output-technology ratio.

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4) Explain the effect of a permanent increase in the share of population engaged in R&D on the growth rate of technology and the level of technology?

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3.9 LET US SUM UP

Endogenous growth models are an important theoretical framework for understanding the growth process. They highlight inter – relationships within the society that helps policy makers. These theories are important because they emphasize that capital accumulation and innovations can induce economic growth, while diminishing returns can reduce it. These models show how long run economic growth can be achieved through spillovers and scale effects of ideas and research within the economy. The models of endogenous growth are primarily concerned with establishing how technological progress can bring about increasing returns to scale. The AK model by Arrow (1962) emphasizes the possibility of productivity depending on output per worker. This implies that technological progress can occur, though unintended, by "learning by doing". As workers continue to specialize in the production process, the productivity of their input will become higher through this specialization. Technological progress in the AK model is modeled as the difference in the initial productivity of the factor before learning by doing and the productivity of the factor after learning by doing – which will be higher. In the AK model, economic growth is induced by savings, capital accumulation, and efficiency. Efficiency is defined as the increase in the productivity of factor inputs by "learning by doing".

The Romer model focuses on the distinction between ideas and objects. The assumptions of the model, yields four equations: (1) Producing output requires knowledge and labor. The production function has constant returns to scale in objects alone, but increasing returns to scale in objects and ideas. (2) New ideas depend on the existence of ideas in the previous period, the number of workers producing ideas, and their productivity. (3) The number of workers producing ideas and the number of workers producing output sums to the population. (4) Some fraction of the population produces ideas. With these equations, Romer model produces the desired long-run economic growth that Solow did not. The Romer model does not have diminishing returns to ideas because they are non-rival.

The Romer model has a *balanced growth path* – on which the growth rates of all endogenous variables are constant and is equal to $g_Y = g_K = g_A = \left(\frac{\dot{A}}{A}\right)^* = \frac{\lambda}{1-\Theta}n$. The long run growth rate of output per worker in this model depends positively on three factors. The parameter λ, the strength of the "standing on shoulders" effect, θ and the growth rate of the number of workers, n. Increase in the level of technology as a result of the permanent increase in R&D has only a level effect on technology. The long-run growth rate of the model returns to the balanced growth path after increasing temporarily.

3.10 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) It uses the assumption that the production function does not exhibit diminishing returns to scale to lead to endogenous growth.

2) The growth rate of output in equation 3.5 depends on the saving rate.

Check Your Progress 2

1) Output Y is produced using labor L_Y and a number of different capital goods, x_i which we call intermediate goods.

2) λ is an index of how slowly diminishing marginal productivity sets in for researchers. θ represents a positive externality associated with the "giant shoulder" effect.

Check Your Progress 3

1) The steady state growth rate of output per worker equals the steady state growth rate of A. That is $g_Y = g_K = g_A$

2) The long run growth rate of output per worker in this model depends positively on three factors. The parameter λ, the strength of the "standing on shoulders" effect, θ and the growth rate of the number of workers, n.

3) Refer to equations (3.31) to (3.47) in the text.

4) A permanent increase in the share of population devoted to research has only a level effect on technology. The level of technology is permanently higher as a result of the permanent increase in R&D. The long-run growth rate of the model returns to the balanced growth path after increasing temporarily.

UNIT 4 BUSINESS CYCLE*

Structure

4.0 OBJECTIVES

After going through the unit you will be able to

- explain the concept and features of Business cycle;
- identify the various phases of Business cycle;
- ascertain the theoretical framework which explains the occurrence of business cycle;
- distinguish between the monetary and real factors behind business cycle; and
- distinguish between the leading, lagging and coincident indicators.

4.1 INTRODUCTION

Rapid economic growth witnessed by many developed economies during the past two centuries has not been a smooth one. There have been periodical ups and downs in the GDP levels of these countries. Along with output, there have been fluctuations in various economic aggregates such as income, employment and prices and their long term trends. These economies have experienced phases of

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expansion and contraction in output and other economic aggregates alternatively. These alternating phases of upswings and downswings are known as business cycles.

Theoretical explanations of business cycles evolved in the early 20^{th} century. Periods of expansion and contraction in an economy exhibited a remarkable degree of regularity. The characteristics of these phases are carefully documented by economists like Wesley Mitchell, Simon Kuznets and Frederick Mills. Mitchell documented the co-movement of variables over the cycles; Mills documented the co-movement of prices and quantities over expansions and contractions, while Kuznets studied the patterns of both growth and fluctuations. The 1930s was a very active period of business cycle research as the National Bureau of Economic Research (NBER) continued its program (begun by Mills and Mitchell) of empirically documenting the features of business cycles. However, interest in business cycles waned after the publication of Keynes' General Theory which turned attention away from Business cycles to short run management of the economy. Interest in business cycles revived in the 1970s when the prevalent economic crisis in many countries could not be explained by Keynesian model.

In this unit we first explain the features of business cycles and the various phases of business cycles. We proceed further to examine how to identify business cycles and measure the aggregate state of the economy using various economic series. Subsequently we explain the important theoretical frameworks of business cycles.

4.2 FEATURES OF BUSINESS CYCLES

Business cycles are economy-wide fluctuations in output, unemployment, prices, revenue, profits, and interest rates, among other variables. These fluctuations occur across the economy and over a number of years. Fluctuations always take place in an economy. Business cycles, however, do not refer to fluctuations that are specific to one geographic region or industry within an economy. To identify business cycles, we must look at factors that can have an effect on the entire economy.

Business cycles consist of recurrent alternating phases of expansions and contractions in a number of economic variables including employment, production, real income, and real sales. Business cycles involve multidimensional processes, in which quantities and prices, stocks and flows, outputs and inputs, real, monetary, and financial variables all tend to move together. These are asymmetric in the sense that expansions typically exceed contractions in size and duration. Business cycles can be distinguished from the other fluctuations in that they are usually *larger, longer,* and *widely diffused.*

The major features of business cycles are as follows:

1) Though business cycles do not show the same regularity, they have some distinct phases such as expansion, peak, recession, trough and recovery. The duration of cycle can vary between two years to twelve years.
2) Business cycles are *synchronic*. Depression or contraction occurs simultaneously in most industries or sectors of the economy. Recession passes from one industry to another and chain reaction continues till the whole economy is in the grip of recession. Similarly, expansion spreads through various linkages between industries or sectors.
3) Fluctuations occur simultaneously in the level of output as well as employment, investment, consumption, etc.
4) Consumption of durable goods and investment are affected the most by cyclical fluctuations. As stressed by Keynes, investment is very unstable as it depends on profit expectations of private entrepreneurs. Any change in these expectations makes investment unstable. Thus the amplitude of fluctuation in the case of durable household effects is higher than that of GDP.
5) Consumption of non-durable goods and services do not vary much during the different phases of business cycles. Past data of business cycles reveal that households maintain a great stability in the consumption of non-durable goods. Thus the amplitude of fluctuations in the case of non-durable consumption goods is lower than that of GDP.
6) The immediate impact of recession or expansion is on the inventories of goods. When recession sets in, inventories start accumulating beyond the desired level. It leads to cut in production of goods. In contrast, when recovery starts, the inventories go below the desired level. It encourages business houses to place more orders for goods which boost production and stimulates investment.
7) Profits fluctuate more than any other type of income as the occurrence of business cycles causes lot of uncertainty for the businessmen and makes it difficult to forecast economic conditions. During depression, profits turn negative and many businesses go bankrupt.
8) Business cycles are international in character. That is, once started in one country, they spread to other countries through contagion effect. The downslide in financial market, for example, in one country spreads rapidly to other country as financial markets are linked globally through capital flows. Further, recessions in one country, say the United States can spread to other country as the imports of the U.S.A. will decline. Countries which are major exporter to the U.S will witness a decline in their exports and may witness recession.

4.3 PHASES OF BUSINESS CYCLES

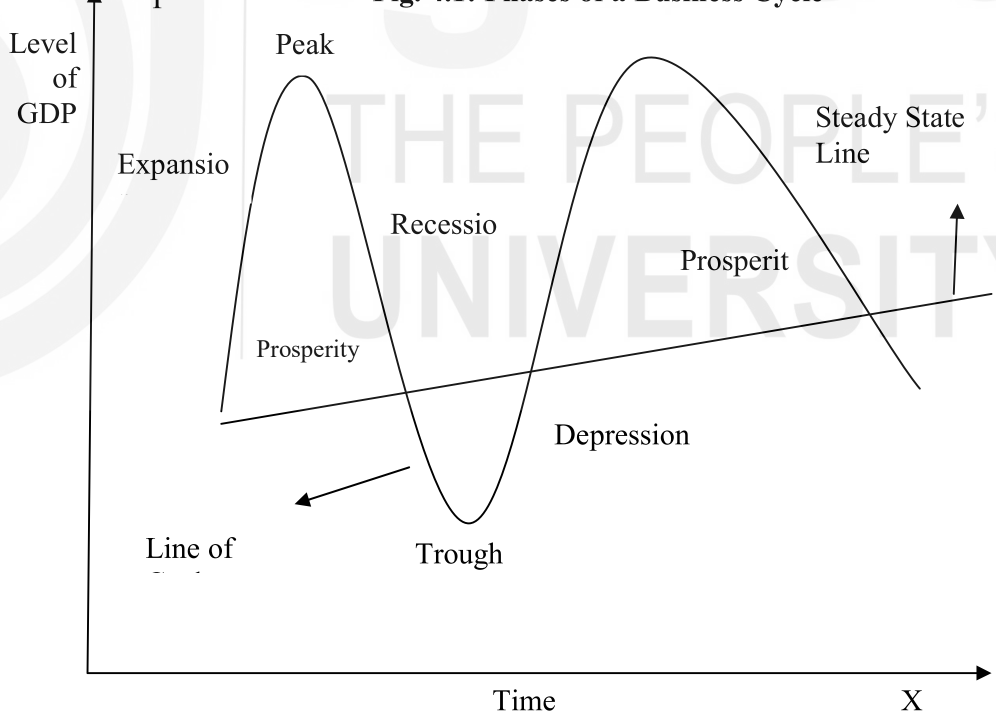
Business cycles are characterized by expansion of economic variables in one period and contraction in the subsequent period. In Fig. 4.1 you can observe the upward sloping curve (expansion phase) there is acceleration in growth rate. The downward sloping segment of the curve indicates the ‘contraction phase’.

In Fig. 4.1 the upward sloping straight line indicates the steady state growth path or the long run growth path of the GDP. The actual GDP fluctuates around the steady state growth path due to business cycles.

According to some researchers there are four phases of a business cycle, viz., expansion, recession, depression and recovery. The four phases of a business cycle are also depicted in Fig. 4.1. In fact, the expansion phase comprises both recovery and expansion. Similarly, the contraction phase consists of both recession and depression. You should note that the difference between recession and depression is one degree. In the recession phase there is a deceleration in the growth rate. In the depression phase, economic growth is below its long run trend and the economy can witness negative growth rate also.

Similarly, the difference between recovery and expansion is one of degree and extent. After negative growth, the economy passes through the recovery phase and then through the expansion phase. The point at which the expansion ends and a recession begin is called ‘peak’ of a business cycle. The point at which a depression ends and recovery begins is called a ‘trough’. Thus peak and trough are ‘turning points’ in a business cycle.

Fig. 4.1: Phases of a Business Cycle



4.3.1 Expansion Phase

In the expansion phase, there is an increase in various economic factors, such as production, employment, output, wages, profits, demand and supply of products, and sales. An expansion stage can begin as the result of many forces, including willingness of financial institutions to lend more and willingness of business houses to borrow more. There is overall optimism in the economy. The expansion phase continues till the economic environment is favourable.

During the expansion phase, the economy often gets overheated in the sense that various constrains and frictions develop in the economy. Wage rate and prices increase much faster than output leading to hike in production cost and decline in profits. The central bank pursues a restrictive monetary policy so that inflation is in under control.

Economic growth in the expansion phase eventually slows down and reaches its peak. During the peak of a business cycle, economic variables such as production, profits, sales and employment are high; but do not accelerate further. There is gradual decrease in the demand for various inputs due to the increase in input prices. The increase in input prices leads to increase in production prices while real income of people does not increase proportionately. It leads consumers to restructure their monthly budget and the demand for products, particularly luxuries and consumer durables, starts falling. The peak also occurs before various economic indicators such as retail sales and the number of employed people falls. When the decline in the demand for products become rapid and steady, recession takes place

4.3.2 Contraction Phase

In recession phase, all the economic variables such as production, prices, saving and investment, starts decreasing. Generally, in the beginning of the downturn, producers are not aware of the decrease in the demand for their products and they continue to produce goods and services. In such a case, the supply exceeds demand and there is accumulation of inventories. Over the time, producers realize that there is an unwanted accumulation of inventories, escalation in production cost, and decline in profits. Such a condition is first experienced by few industries and slowly spreads to the whole economy. During the recession phase, producers usually avoid new investments which lead to the reduction in the demand for factors of production, and consequent decline in input prices and unemployment. Firms reduce levels of production and the number of people on their payrolls. A chain reaction starts, lower income, lower demand, lower output, lower employment, and so on. The adverse effects of recession extent beyond the purely economic realm and influence the social fabric of society as well. Social unrest and crimes tend to rise during recession.

When recession continues further, economic growth rate may be negative also. This phase is sometimes termed as 'depression'. During depression, there is not just a decline in the growth rate; there is a decline in the absolute level of GDP. As sales declines, business houses find it difficult to repay their debts. As business sentiments are low enough to carry out new investments, demand for credit declines. Banks also become cautious in their lending as the chances of default on repayment increases. The economy however revives its growth rate over a period of time and optimism build up in certain sectors of the economy. This leads to reversal of the recession phase and the recovery phase starts.

Individuals and organizations start developing a positive attitude towards the various economic factors, such as investment, employment and production. In the recovery phase, there is an increase in consumer spending and demand for consumer goods. This provides incentive to firms to increase production, carry out new investments, hire more labour, etc. Further, there could be some investment during the recession phase due to replacement of obsolete machines and maintenance of existing capital stock.

Price level plays a very important role in the 'recovery phase' of an economy. As pointed out earlier, during the recession phase decline in input prices is greater than the decline in product prices. This leads to a reduction in the cost of production and increase in profits. Apart from this, in the 'recovery phase, some of the depreciated capital goods are replaced by producers and some are maintained by them. As a result, investment and employment by organizations increases. As this process gains momentum, an economy again enters into the phase of expansion. Thus, the business cycle gets completed.

Check Your Progress 1

1) What is meant by business cycle?

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2) Point out the important features of business cycle.

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3) What are the different phases of business cycle?

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4.4 IDENTIFICATION OF BUSINESS CYCLES

Understanding the various phases of business cycles is essential, because it will help the government in taking counter-cyclical measures. This requires identifying the turning points of a business cycle. In the United States, the National Bureau of Economic Research (NBER) has a dedicated research programme for identifying the dates of business cycle turning points.

Similarly the Euro Area Business Cycle Dating Committee of the Centre for Economic Policy Research (CEPR) identifies the chronology of recessions and expansions of the Euro Area member countries. In India also there have been some attempts by scholars to identify the chronology of business cycles (See, for example, Dua and Banerjee (2000) and Chitre (2001)).

The NBER's Business Cycle Dating Committee maintains a chronology of the United States business cycle. The chronology comprises alternating dates of peaks and troughs in economic activity. A recession is a period between a peak and a trough, and an expansion is a period between a trough and a peak. According to NBER a recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales. Similarly, during an expansion, economic activity rises substantially, spreads across the economy, and usually lasts for several years. Thus, the NBER approach identifies cycles as recurrent sequences of alternating phases of expansion and contraction in the levels of a large number of economic time series. This working definition of business cycle has been in use at the NBER for over fifty years, and it is currently employed by the NBER to identify and date the United States business cycle. These dates are widely accepted by government, researchers and business analysts.

In both recessions and expansions, brief reversals in economic activity may occur – a recession may include a short period of expansion followed by further decline; an expansion may include a short period of contraction followed by further growth. The Business Cycle Dating Committee applies its judgment based on the above definitions of recessions and expansions and has no fixed rule to determine these upturns and downturns.

The Committee does not have a fixed definition of economic activity. It examines and compares the behaviour of various measures of broad activity: real GDP measured on the product and income sides, economy-wide employment, and real income. The Committee also may consider indicators that do not cover the entire economy, such as real sales and the Federal Reserve's index of industrial production (IIP).

Table 4.1: NBER Chronology for United States Business Cycles

(duration in number of months)

Peak month	*Trough month*	*Peak month number*	*Trough month number*	*Duration, peak to trough*	*Duration, trough to peak*	*Duration, peak to peak*	*Duration, trough to trough*
	December 1854	---	660	---	---	---	---
June 1857	December 1858	690	708	18	30	---	48
October 1860	June 1861	730	738	8	22	40	30
April 1865	December 1867	784	816	32	46	54	78
June 1869	December 1870	834	852	18	18	50	36

October 1873	March 1879	886	951	65	34	52	99
March 1882	May 1885	987	1025	38	36	101	74
March 1887	April 1888	1047	1060	13	22	60	35
July 1890	May 1891	1087	1097	10	27	40	37
January 1893	June 1894	1117	1134	17	20	30	37
December 1895	June 1897	1152	1170	18	18	35	36
June 1899	December 1900	1194	1212	18	24	42	42
September 1902	August 1904	1233	1256	23	21	39	44
May 1907	June 1908	1289	1302	13	33	56	46
January 1910	January 1912	1321	1345	24	19	32	43
January 1913	December 1914	1357	1380	23	12	36	35
August 1918	March 1919	1424	1431	7	44	67	51
January 1920	July 1921	1441	1459	18	10	17	28
May 1923	July 1924	1481	1495	14	22	40	36
October 1926	November 1927	1522	1535	13	27	41	40
August 1929	March 1933	1556	1599	43	21	34	64
May 1937	June 1938	1649	1662	13	50	93	63
February 1945	October 1945	1742	1750	8	80	93	88
November 1948	October 1949	1787	1798	11	37	45	48
July 1953	May 1954	1843	1853	10	45	56	55
August 1957	April 1958	1892	1900	8	39	49	47
April 1960	February 1961	1924	1934	10	24	32	34
December 1969	November 1970	2040	2051	11	106	116	117
November 1973	March 1975	2087	2103	16	36	47	52
January 1980	July 1980	2161	2167	6	58	74	64
July 1981	November 1982	2179	2195	16	12	18	28
July 1990	March 1991	2287	2295	8	92	108	100
March 2001	November 2001	2415	2423	8	120	128	128
December 2007	June 2009	2496	2514	18	73	81	91
February 2020	---	---	---	---	128	146	---

You can observe from Table 4.1 that the duration of a cycle is not uniform (see from trough to trough or from peak to peak).

Secondly, the duration of peak to trough (contraction phase) has been shorter than the duration of trough to peak (expansion phase).

4.5 BUSINESS CYCLE INDICATORS

As you already know, a major objective of macroeconomic policy is to maintain stability in economic growth and price level. An important part of the job of the central bank is therefore to gather information of the current and if possible, future economic conditions. The theoretical concept of measuring current business activities using economic series such as GDP, sales, investment, stock prices, etc. is rather simple though its practical application is difficult. Usually, the time pattern of these fluctuating economic series is diverse. While some economic series are expanding at a given point in time, others have already reached their upper turning point (peak) and still others are on the downswing; a few economic activities might even be at a lower turning point (trough). Thus the question is how to measure the overall state of the economy using these economic variables as they have diverse trends.

Economic indicators were conceived at the NBER originally by W.C. Mitchell and A. F. Burns in the 1930s. This approach requires monitoring of economic variables that tend to be sensitive to cyclical changes no matter what their cause. There could be three scenarios: (i) certain economic variables move ahead of business cycles (they 'lead' a business cycle), (ii) certain other economic variables lag behind a business cycle (the turning points in these variables take place later that with certain 'lag'), and (iii) there are still other economic variables which 'coincide' with business cycles. Burn and Mitchell studied a group of about 487 variables to see if the turning points in the variables persistently led, coincided with, or lagged behind the turning points in the U.S. business cycle. Seventy one series were chosen and arranged according to the average lead or lag with regard to the reference revivals. For example, six time series had no average lead or lag. On the average, the leading series were from one to ten months ahead of the reference revivals. The lagging series were on the average from one to twelve months behind.

According to Business Cycle Indicators Handbook 2020, a business cycle indicator should fulfil the following criteria:

(i) Conformity: the series must conform well to business cycles;

(ii) Consistent Timing: the series must exhibit a consistent timing pattern over time as a leading, coincident or lagging indicator;

(iii) Economic Significance: the cyclical timing of the series must be economically logical;

(iv) Statistical Adequacy: data on the variable must be collected and processed in a statistically reliable way;

(v) Smoothness: month-to-month movements in the variable must not be too erratic; and

(vi) Currency: Data on the variable must be available on a reasonable prompt schedule.

Those series are selected which are similar in timing at peaks and troughs with business cycles. Business cycle indicators are classified into three groups, viz., leading, roughly-coincident and lagging.

4.5.1 Leading Indicators

Leading economic indicators help us assess where the economy is headed. They foreshadow what is coming, such a turning point, before it actually happens.

One of the most significant leading indicators is the stock market itself, gauged by an index such as the S&P 500. It will begin to rise before economic environment seems favourable, and it will begin to decline before economic conditions seem to warrant it. Another important leading indicator is interest rates. Low interest rate stimulates borrowing and buying, which favours the economy. An increase in interest rates shows the economy is doing well, but eventually rising interest rates lead to a slowdown because less people borrow money to start new projects.

4.5.2 Lagging Indicators

Unlike leading indicators, lagging indicators turn around after the economy changes. Although they do not typically tell us where the economy is headed, they indicate how the economy changes over time and can help identify long-term trends. Lagging economic indicators reveal past information about the economy.

Gross Domestic Product (GDP) is how much a country is producing. There is significant lag time between when the data is compiled and when it is released, yet it is still an important indicator. Many consider a recession to be underway if two quarters see back-to-back declining GDP. Other indicators, such as the Consumer Price Index (CPI), are also sometimes considered lagging indicators, since they reveal information that is already known to most consumers.

4.5.3 Coincident Indicators

Coincident indicators change (more or less) simultaneously with general economic conditions and therefore reflect the current status of the economy. They give consumers, business leaders, and policy makers an idea about where the economy is currently, right now. When the economy rises today, then coincident indicators are also rising today. Similarly if the economy declines today, then coincident indicators are also declining today. Typical examples of coincident indicators are industrial production or turnover. In Table 4.1 we present a list of business cycle indicators.

Table 4.2: Business Cycle Indicators

Leading	Roughly-Coincident	Lagging
I. Investment in Fixed Capital and Inventories		
New building permits; housing starts; residential fixed investment; New business formation; New capital appropriations; Contracts and orders for plant and equipment; Change in business inventories	Production of business Equipment; Machinery and equipment sales	Backlog of capital appropriations; Business expenditures for new plant and equipment; Trade inventories
II. Consumption, Trade, Orders, and Deliveries		
New orders for consumer goods and materials; Change in unfilled orders, durable Goods; Vendor performance (speed of deliveries); Index of consumer sentiment	Production of consumer Goods; Trade sales	
III. Employment, Production, and Income		
Average workweek; overtime Hours; Accession rate; layoff rate; New unemployment insurance claims; Productivity (output per hour); Rate of capacity utilization	Non-agricultural employment; unemployment rate; GDP; personal income; industrial production	Average duration of unemployment; Long term unemployment
IV. Prices, Costs, and Profits		
Bond prices; Stock prices; Sensitive materials prices; Profit margins; Total corporate profits; net cash flows		Unit labour costs; Labour share in national income

V. Money, Credit, and Interest		
Monetary growth rates; Change in liquid assets; Change in consumer credit; Total private borrowing; Real money supply	Velocity of money	Short-term interest rates; Bond yields; Consumer credit outstanding; Commercial and industrial loans outstanding
Note: The selection is based on the U.S. indicators published in *Business Conditions Digest,* a monthly report by the Bureau of Economic Analysis, U.S. Department of Commerce		

Check Your Progress 2

1) What are the criteria that form the basis for selection of a business cycle indicator?

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2) What is the importance of a lagging indicator?

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4.6 THEORIES OF BUSINESS CYCLES

We have explained above the various phases and common features of business cycles. Now, an important question is what causes business cycles. Several theories of business cycles have been propounded from time to time. Each of these theories spells out different factors which cause business cycles.

4.6.1 Keynes' Theory of Business Cycle

J.M. Keynes in his seminal work 'General Theory of Employment, Interest and Money' made an important contribution to the analysis of the causes of business cycles. According to Keynes, the changes in the level of aggregate effective demand will bring about fluctuations in the level of income. The aggregate demand is composed of demand for consumption goods and demand for investment goods. According to Keynes, propensity to consume is more or less stable in the short run. Private investment however depends upon profit motive and business expectations about the economy. Thus fluctuation in aggregate demand depends primarily upon fluctuations in investment demand. Multiplier plays a significant role in causing magnified changes in income following a reduction or increase in investment.

Keynesian theory however fails to explain the cumulative character of business cycle. For example, suppose that investment rises by 100 rupees and that the magnitude of multiplier is 4. From the theory of multiplier, we know that national income will rise by 400 rupees and if multiplier is the only force at work that will be the end of the matter, with the economy reaching a new stable equilibrium at a higher level of national income. But in real life, this is not likely to be so, for a rise in income produced by a given rise in investment will have further repercussions in the economy. This reaction is described in the 'principle of accelerator' (accelerator is the impact of income on investment). Samuelson combined the accelerator principle with the multiplier and showed that the interaction between the two can bring about cyclical fluctuations in economic activity.

4.6.2 Schumpeter's Innovation Theory of Business cycles

Joseph Schumpeter considered trade cycles to be the result of innovation activity of the entrepreneurs in a competitive economy. Schumpeter calls the equilibrium state of the economy as a "circular flow" of economic activity which just repeats itself period after period. The circular flow of economic activity gets disturbed when an entrepreneur successfully carries out an innovation. According to Schumpeter, the primary function of an entrepreneur is innovation activity which yields him/ her real 'profit'.

According to Schumpeter, introduction of a major innovation leads to a business cycle. As the innovator-entrepreneur begins bidding away resources from other industries, money incomes increase and prices begin to rise thereby stimulating further investment. As the innovation steps up production, the circular flow in the economy swells up. Supply exceeds demand. The initial equilibrium is disturbed. There is a wave of expansion of economic activity. This is what Schumpeter calls the "primary wave". This primary wave is followed by a "Secondary wave" of expansion. This is due to the impact of the original innovation on the competitors. You can imagine the impact of innovation if you relate it to some real life examples such as the Internet, mobile phone, and on-line transactions.

As the original innovation proves profitable, other entrepreneurs follow it in "swarm-like clusters". Innovation in one sector induces innovations in related sectors. Money incomes and prices rise. As potential profits in these industries increase, a wave of expansion in the whole economy follows.

This period of prosperity ends as soon as 'new' products induced by the waves of innovations replace old ones. Since the demand for the old products goes down, their prices fall and consequently their producer-firms are forced to reduce their output. When the innovators begin repaying their bank loans out of the newly-earned profits, the quantity of money in circulation is reduced as a result of which prices tend to fall and profits decline.

In this atmosphere, uncertainty and risks increase. Recession sets in. The economy cannot continue in recession for long. Entrepreneurs continue their search for profitable innovations. The natural forces of recovery bring about a revival.

4.6.3 Samuelson's Model of Business Cycles: Interaction between Multiplier and Accelerator

Samuelson in his seminal paper convincingly showed that an autonomous increase in the level of investment raises income by a magnified amount depending upon the value of the multiplier. This increase in income further induces the increases in investment through acceleration effect. The increase in income brings about increase in aggregate demand for goods and services. To produce more goods we require more capital goods for which extra investment is undertaken. Thus the relationship between investment and income is one of mutual interaction; investment affects income which in turn affects investment demand and in this process income and employment fluctuate in a cyclical manner.

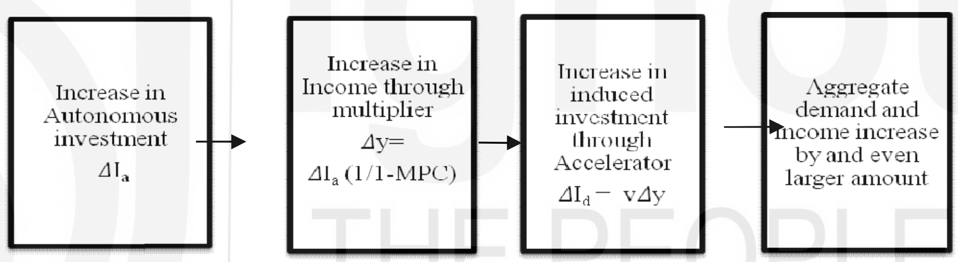


Fig. 4.2 shows how income and output will increase by even larger amount when accelerator is combined with the Keynesian multiplier.

ΔI_a = Increase in Autonomous Investment

Δy = Increase in Income

$\frac{1}{1-MPC}$ = Size of multiplier when MPC = Marginal Propensity to Consume

ΔI_d = Increase in Induced Investment

v = Size of Accelerator

Let us assume that there is an increase in investment in the economy. This will result in a magnified increase in output and income due to multiplier effect. When output increases under the influence of multiplier effect, it induces further increase in investment. The extent to this induced investment in capital goods industries depends on the capital-output ratio (v).

Increase in investment leads to further increase in income, which again leads to increase in investment. The pattern of the interaction between multiplier and accelerator however differs depending upon the magnitudes of the marginal propensity to consume and capital-output ratio. It implies that the interaction between multiplier and accelerator can give rise to business cycles.

The model of interaction between multiplier and accelerator can be mathematically represented as under:

$$\boldsymbol{Y_t = C_t + I_t} \qquad ...(4.1)$$

$$\boldsymbol{C_t = C_a + c\,(Y_{t-1})} \qquad ...(4.2)$$

$$\boldsymbol{I_t = I_a + v\,(Y_{t-1} - Y_{t-2})} \qquad ...(4.3)$$

where Y_t , C_t , I_t stand for income, consumption and investment respectively for period t, C_a stands for autonomous consumption, I_a for autonomous investment, c for MPC and v for capital-output ratio or accelerator.

From the above equation it is evident that consumption in a period t is a function of income of the previous period, Y_{t-1}. That is, one period lag has been assumed for income to determine the consumption of a period. As regards induced investment in period t, it is taken to be a function of the change in income in the previous period. It means that there are two period gaps for changes in income to determine induced investment. In the equation (4.3) above, induced investment equals $v\,(Y_{t-1} - Y_{t-2})$. Substituting equations (4.2) and (4.3) in (4.1), we have the following:

$$\boldsymbol{Y_t = C_a + c\,(Y_{t-1}) + I_a + v\,(Y_{t-1} - Y_{t-2})} \qquad ...(4.4)$$

Equation (4.4) indicates how changes in income are dependent on the values of MPC (c) and capital-output ratio v, (i.e., accelerator).

By taking different combinations of the values of c and v, Samuelson could describe different paths which the economy would follow. The various combinations of the values of c and v are shown in Fig. 4.3.

Fig. 4.3: Combinations of c and v

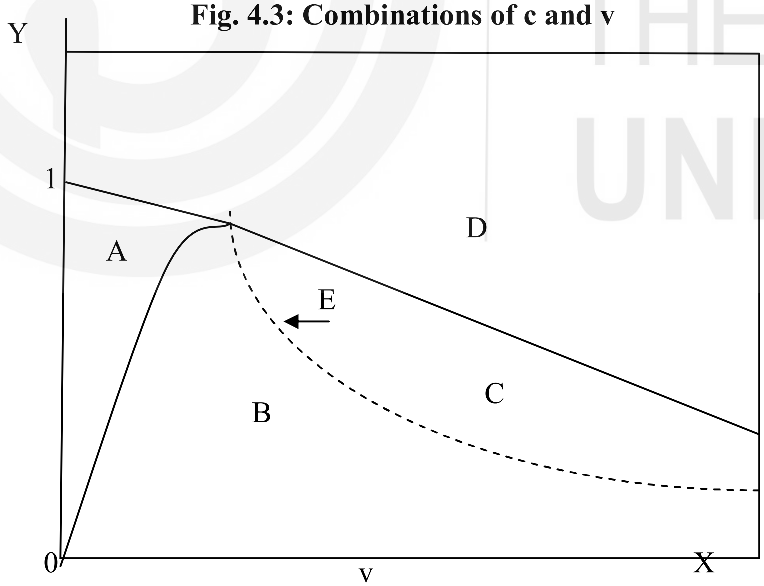
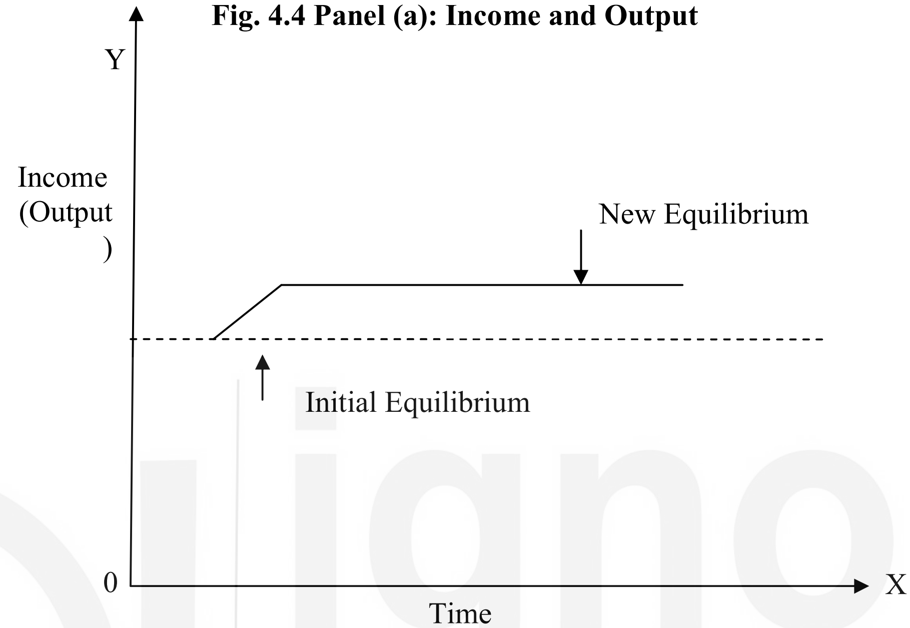


Fig. 4.3 shows the four paths which the economic activity can have depending upon combinations of the values of marginal propensity to consume (c) and capital-output ratio (v).

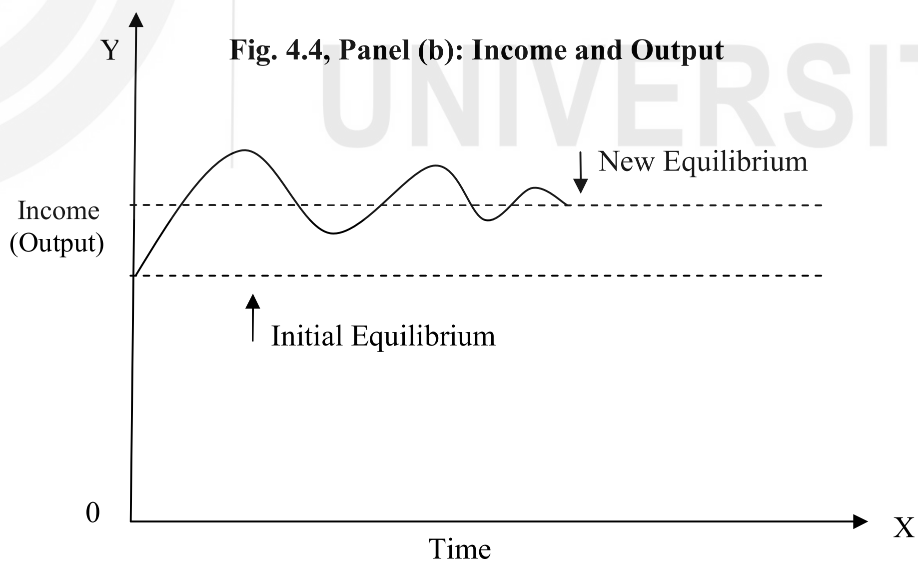
The five paths or patterns of movements in output or income can have depends upon the combinations of the values of c and v. We depict these paths in Fig. 4, Panels (a) to (e). When the combinations of c and v lie in the region marked A, an increase in investment will increase output a decreasing rate. Finally it reaches a new equilibrium as shown in panel (a) of Fig. 4.4.

Fig. 4.4 Panel (a): Income and Output



If the values of c and v lie in region B of Fig. 4.3, a change in investment will generate fluctuations in income which follow the pattern of a series of damped cycles as shown in panel (b) of Fig. 4.4. It means that the amplitude goes on declining until the cycles disappear over a period of time.

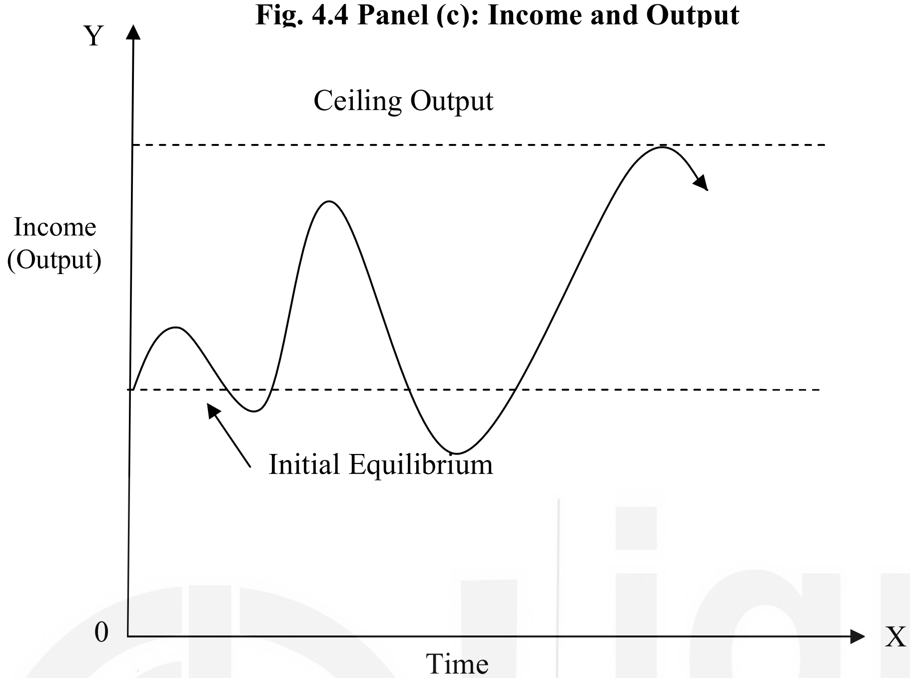
Fig. 4.4, Panel (b): Income and Output



You should note that region C of Fig. 4.3 represents the combinations of c and v which are relatively high as compared to the region B. Such values of multiplier and accelerator bring about explosive cycles as given in panel (c) of Fig. 4.4.

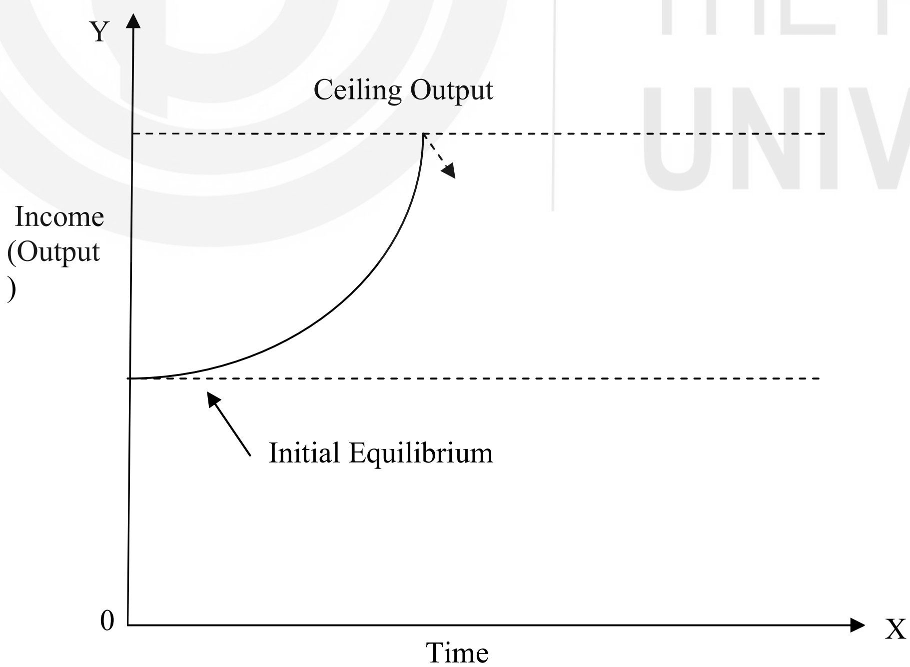
It implies that the fluctuations of income will be successively greater and greater in amplitude.

Fig. 4.4 Panel (c): Income and Output



The region D of Fig. 4.3 describes the combinations of c and v which cause income to move upward or downward at an increasing rate. We have depicted it in panel (d) of Fig. 4.4.

Fig. 4.4 Panel (d): Income and Output



In a special case when values of c and v lie in the region E of Fig. 4.3, they produce fluctuations in income of constant amplitude and are shown in panel (e) of Fig. 4.4. You should note that all the above five cases do not give rise to cyclical fluctuations or business cycles. It is only combinations of c and v lying in the regions B, C and E that produce business cycles.

Fig. 4.4 Panel (e): Income and Output

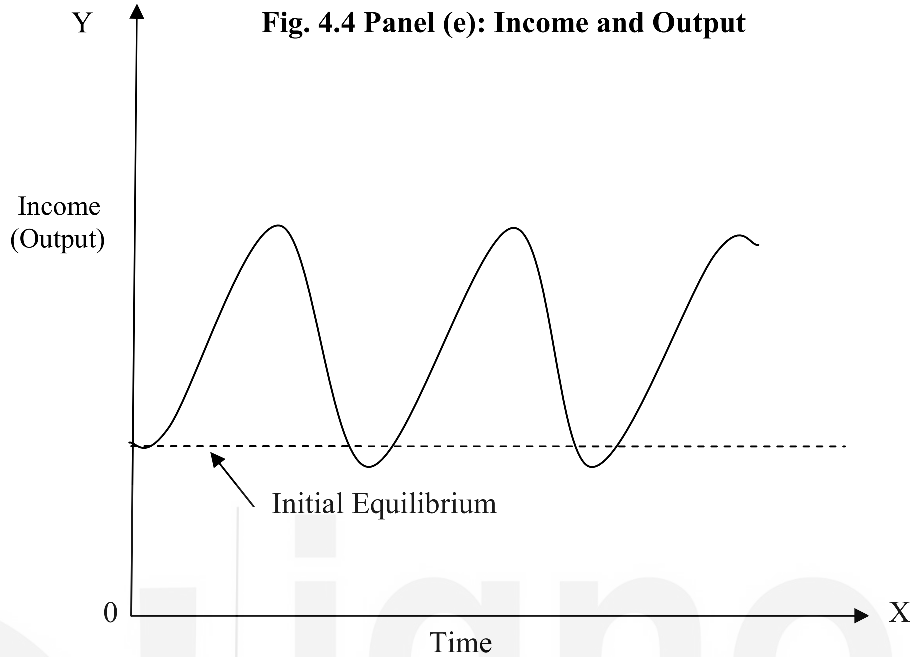


Fig. 4.4 Panel (a) to (e) shows different patterns of income (output) movements for various values of c and v which respectively determine the magnitudes of multiplier and accelerator.

4.6.4 Real Business Cycle Theory

According real business cycle theory, monetary shocks or expectation changes have no role to play in a business cycle. The real business cycle theory makes the fundamental assumption that root cause of business cycle is real shocks to an economy. These shocks could be from the supply side such as technology shocks (changes in total factor productivity). Technological shocks include innovations, bad weather, stricter safety regulations, etc.

Business cycles are primarily caused by real or supply side shocks that involve exogenous large random changes in technology. An initial shock in the form of technical progress shifts the production function upward. This leads to increase in available resources, investment, consumption and real output. With the increase in investment, the capital stock increases which further increases real output, consumption and investment. This process of expansion of the economy continues erratically due to changes in technology over time.

Real business cycle theory explains the causes of recession as follows: A recession in the real business theory is just the reverse of expansion. Negative real shocks decreases the available resources, and shifts the production function downward as a result of which output declines.

There could be several examples of negative real shocks such as decline in technology (i.e., technical regress), unexpected rise in input prices (crude oil crisis), scanty rainfall (severe drought), etc. This starts a process of decline in investment, consumption, output and employment. But the models of real business cycle do not explain a recession.

Check Your Progress 3

1) Explain in brief how multiplier and accelerator interact to generate business cycles.

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2) Explain the real business cycle theory.

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4.7 LET US SUM UP

In this unit we focussed on three issues: characteristics of business cycle, indicators of business cycle, and some important theories of business cycle. Business cycle should be thought of as apparent deviations from a trend in which many economic variables move together. The fluctuations are typically irregularly spaced and of varying amplitude and duration. Nevertheless, the one very regular feature of these fluctuations is the way variables move together. Business cycle is characterised by four phases, viz., expansion, recession, depression and recovery.

A major problem in empirical identification of business cycle is the lack of a single and consistent measure of aggregate economic activity. In view of this, movements in a number of indicators are considered for identification of turning points of a business cycle. Timing and amplitude of these variables are used to group them into leading, lagging and coincident indicators.

Some of the important theories which explain these feature of business cycles are: (a) Keynesian theory which showed that changes in the level of aggregate effective demand bring about fluctuations in the level of income, output and employment; (b) Samuelson model of business cycle which shows that the interaction between multiplier and accelerator gives rise to cyclical fluctuations in economic activity; and c) real business cycle theory which says that business cycles are due to real shocks to an economy.

4.8 ANSWERS/ HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Business cycle consists of recurrent alternating phases of expansion and contraction in a large number of economic activities.
2) Business cycles are periodic, synchronic and once they start in one country, they spread to other countries through trade relations between them. See Section 4.2 for details.
3) There are four phases of a business cycle, viz., expansion, recession, depression and recovery. See Section 4.3 for details.

Check Your Progress 2

1) A business cycle indicator should fulfil six criteria as described in Section 4.5.
2) The importance of a lagging indicator is its ability to confirm that a pattern is occurring. Unemployment is one of the most popular lagging indicators. If the unemployment rate is rising, it indicates that the economy has been doing poorly.

Check Your Progress 3

1) You should describe equation (4.4) and draw inferences on the basis of Fig. 4.3.
2) According to real business cycle theory, supply shocks generate business cycles. Refer to Section 4.6.4.